Influence of Gas Species on

Thermophoretic Velocity of PMMA Particle

(PMMA 粒子の熱泳動速度に気体種が及ぼす影響)

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Dedication

This work is dedicated to my wonderful parents, Razali Bin Lazim and Rahemah Binti Abdul Rahim.

I also dedicate this work to my beloved wife, Nurmazeha Binti Mail, who have provided never-ending support and encouragement from day one.
In the temperature field, a small particle will move towards the lower temperature side. This phenomenon is called thermophoresis, which influences the movement of soot particles in exhaust gas from combustors. In recent years, our group has developed a device for conducting experiments repeatedly under a microgravity environment in a very short period time, i.e. 0.3 s, by means of the free-fall method, to accumulate data of the thermophoretic velocity.

Considering the environment of soot, it is indispensable to understand quantitatively the phenomenon for a gas mixture. However, before analyzing the phenomenon for the gas mixture, it would be desirable also to know the characteristics of the phenomenon for each gas component, since the combustion gas mixture contains many gas species. In the present work, the investigations have been carried out to examine the influence of gas species on thermophoretic velocity of particle.

As the first step, the characteristics of the thermophoresis for several pure gases, i.e., argon, nitrogen, and carbon dioxide are experimentally examined. Particles adopted are PMMA spheres of 2.91 μm in mean diameter. The temperature gradient is set at 10 K/mm, and the pressure is set at several conditions in the range from 20 kPa to 100 kPa. Obtained experimental results are compared with theoretical predictions; a notable discrepancy is found for carbon dioxide, while the results for other two gases are
consistent with predictions. Some attempts are made to fix the discrepancy: first by modifying constants and second by modifying two empirical coefficients in the theory.

In the next step, an attempt is made to investigate dependence of thermophoretic parameters on gas properties. Atmospheric gases chosen are pure gases of argon, nitrogen, carbon dioxide, methane, and nitrous oxide. The residue values for all combinations of the coefficients are calculated from the obtained experimental results; several combinations of the coefficients are possible to fit the prediction with the experimental results. The tangential momentum accommodation coefficient is estimated by assuming the thermal accommodation coefficient to be unity. Physical relationships between the coefficient and gas properties are investigated.

In the final step, the thermophoretic velocity is experimentally investigated for several gas mixtures, i.e., argon-nitrogen, argon-carbon dioxide, and nitrogen-carbon dioxide. The temperature gradient and the pressure are 60 K/mm and 70 kPa, respectively. The tangential momentum accommodation coefficient is estimated from experimental result by assuming the thermal accommodation coefficient at unity, and it is compared with predictions calculated from values of composing pure gases by means of some methods given in references; among those methods, the intermolecular-interaction-based method is found to be the best for all mixture conditions.
Acknowledgement

In the name of Allah, Most Gracious and Most Merciful

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I also wish to thank the Ministry of High Education, Malaysia and University Tun Hussein Onn Malaysia, Malaysia for sponsoring my study.

On top of all, I owe a particular debt of gratitude and appreciation to my wonderful parents for their long-distance support. My deepest gratitude I must reserve for my beloved wife Nurmazeha Binti Mail and my son Irsyad Zafran Bin Mohd Azahari, whose I am very thankful for patience and understanding. They never uttered any words of discouragement and always inspired me to carry out the thesis work with sincerity.

Thank you,

Mohd Azahari Bin Razali
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Chapter 1: Introduction

Chapter 1

Introduction

1.1 Background

The improvement in technology produces development in transportation and industrial such that our lives become convenient. However this improvement requires increment in the energy consumption. Since 1970, the world energy consumption growth is about 2.3% per year [1]. It is estimated about 86% of the energy source is produced from fossil fuels such as coal, petroleum, and natural gas. Fossil fuels are of great importance since they can produce significant amount of energy per unit weight. Consumptions of all these three sources have been increased such that the reserves of these sources have been used up quickly. Some developments have been done to produce alternative energy from the non-fossil sources such as nuclear and biomass energies. Some of these developments have reached the practical stage but only 16% of the global final energy consumption comes from these sources. Currently, fossil fuels are still considered as valuable energy sources; which is important to be used efficiently.

Significant amount of heat energy is released due to the combustion of fossil fuels. However, the use of fossil fuels raises the environmental concerns. Burning of fossil fuels produces carbon dioxide, which is one of the greenhouse’s gases that enhances radiative forcing, which contributes to global warming. It is important to understand the
combustion phenomenon in a combustion chamber since improvement of the energy efficiency will produce undesirable gas emissions and environmental pollution.

Soot is one of the main pollutants emitted from the combustion devices. It is important to understand the behavior of soot particles in the combustion field for emission control. Dobashi et al. [2] have conducted experiments to examine smoke generation process in a flickering pool fire and have found the residence time of soot particle in the soot production is increased by the flickering motion of the flame. It is noted that the thermophoretic force might increase the residence time in the soot production region. Quantitative understanding of the phenomenon is indispensable for controlling the movement of particles in this system.

There have been some direct measurements on the thermophoretic phenomenon of such an aggregate particle. Zheng and Davis [3] have measured the thermophoretic force acting on an aggregate of polystyrene latex spheres, and found that the force is affected by the number of primary spheres in the aggregate. Suzuki and Dobashi [4] have conducted direct measurements on the thermophoretic velocity of the soot particles, and have revealed that the velocity is dependent not only on the macroscopic size of the soot particle but also on the aggregation condition; experimental results suggest that this phenomenon is dominated by the size of primary spheres when the aggregation is coarse. Thus, prior understanding for an aggregate, well knowledge about the phenomenon for a single sphere is important.
1.2 Existing Theory of Thermophoresis

Thermophoresis is the phenomenon that a small particle in a gas with a temperature gradient moves toward the lower temperature side. This phenomenon is caused by the momentum exchange between the gas molecules and the particle. The primary mechanism of this exchange is the gas molecules collide from the higher temperature side and later transfer negative momentums to the particle with amount that exceed those of positive momentums transferred by the gas molecules colliding from the lower temperature side. As the result of this momentum exchange, both the particle and the surrounding gas move in mutual opposite direction. The flow of the gas is referred to as the superficial slip flow.

There are several theoretical works have been carried out to study the thermophoretic phenomenon of a spherical particle. Theories of this phenomenon are derived by considering the boundary condition on the surface of a single spherical particle suspended in a gas with a temperature gradient. Brock [5] derived a theoretical solution to the thermophoretic force on the particle by applying a slip boundary condition on its surface, which includes effects of the thermal slip, the viscous slip, and the temperature jump. The equation is shown as below:

\[
F_T = -\frac{9\pi\mu^2 d}{2\rho T_{F0}} \frac{(k + C_T Kn)}{(1 + 2 C_M Kn)(1 + 2k + 2 C_T Kn)} |\nabla T|,
\]

where \(\mu\), \(d\), \(|\nabla T|\), \(\rho\), \(T_{F0}\), \(k\), \(Kn\), \(C_M\), and \(C_T\) are the viscosity, the particle diameter, the temperature gradient, the density of the surrounding gas, the reference temperature, the gas-to-particle thermal conductivity ratio, Knudsen number, constants for slip flow, and temperature jump, respectively. Here, the reference temperature \(T_{F0}\) is defined as the supposed gas temperature at the center of the particle in the given temperature field.
without the existence of the particle, and Knudsen number $Kn$ is the ratio of the mean-free-path $l$ to the particle radius. The mean-free-path is calculated from the following equation [6]:

$$l = \frac{\mu}{0.499P} \sqrt{\frac{\pi RT_0}{8}}. \quad (1-2)$$

where $P$ and $R$ are the pressure and the gas constant, respectively.

Derjaguin and Yalamov [7] derived the thermophoretic velocity from the Brock’s theoretical solution by equating the balance between the thermophoretic and the drag forces:

$$F_T + F_D = 0. \quad (1-3)$$

Here, the drag force $F_D$ is given as follows:

$$F_D = \frac{3\pi \mu d V_T}{C_C}, \quad (1-4)$$

where $V_T$ and $C_C$ are the thermophoretic velocity and Cunningham’s correction factor, respectively. The slip correction factor adopted is $C_C = 1 + AKn$, where $A = 1.257 + 0.4 \exp(-1.10/Kn)[8]$. Thus, the equation of thermophoretic velocity is shown as below:

$$V_T = \frac{3}{2} \frac{\mu C_C}{\rho_g T} \left(\frac{k + C_T Kn}{1 + 2C_M Kn (1 + 2k + 2C_T Kn)}\right)|\nabla T|. \quad (1-5)$$

Hoshino et al. [9] have derived an improved theoretical solution of the thermophoretic velocity by applying the boundary condition proposed by Lockerby et al. [10], which includes the thermal stress slip and the higher order isothermal slip. The equation is given as follows:
Chapter 1: Introduction

\[ V_T = \frac{3 \mu C_c}{2 \rho T_0} \left[ \frac{(k + C_f Kn) + \frac{4}{3} (C_m C_f Kn^2 + C_m Kn[k - 1])}{1 + 3C_m Kn + \frac{9}{2\pi} Pr Kn^2 (1 - \gamma^{-1})} \right] [\nabla T], \]  

(1 – 6)

where \( Pr \) and \( \gamma \) are Prandtl number and the specific heat ratio, respectively. Detail of the derivation of thermophoretic velocity is shown in Appendix A.

1.3 Previous Experiments of Thermophoresis

Several experiments have been performed to measure the thermophoretic force or the thermophoretic velocity. Fredlund [11] attempted a systematic experiment to examine the effect of the temperature field upon a disk suspended on a balance. The experiments have been performed with various gases: hydrogen, argon, nitrogen, oxygen, and air. It is shown that at the given distance between the plates with different temperatures, the thermophoretic force is a linear function of pressure.

The thermophoretic force and the thermophoretic velocity are measured by several methods: Millikan cell [12 – 15], electrodynamic balance [16, 17], precipitation in a thermoprecipitator [18, 19], jet technique [8, 20, 21], and deflection of a particle suspended by a small wire [22, 23].

Above experimental methods are complex in practical implementation, and as a consequence, involve numerous errors. Among those, errors caused by buoyancy are the largest problem; in a field with a temperature gradient, buoyancy induces natural convection, which influences the movement of particles and disturbs the measurement. The velocity of such natural convection is usually comparable to the thermophoretic velocity and cannot be measured directly. To avoid this problem, some experiments have been conducted under microgravity conditions. Toda et al. [24, 25] performed
experiments in a drop tower facility and demonstrated that the microgravity environment satisfactorily suppresses the disturbance. Prodi et al. [26, 27] also conducted microgravity experiments by means of a drop tower facility and/or parabolic flights. However, their reported data still seem to contain some errors, possibly owing to limited trial numbers of experiments, so that those data are not sufficient enough to make a quantitative comparison with theories [28].

1.4 Motivation of Present Work

Recently, our group has developed a device for conducting experiments repeatedly under a microgravity environment in a very short period of time, i.e. 0.3 s, by means of the free-fall method, to accumulate data of the thermophoretic velocity. It has been confirmed that satisfactory accuracy can be attained if the amount of data is sufficient for statistical treatment [28]. By comparing the obtained experimental results with the existing theory as shown in Eq. (1-5), a notable difference is found for particles with high thermal conductivity [29].

The problem of the difference has been solved by reconsidering the boundary condition to improve the theory as shown in Eq. (1-6). Figure 1.1 shows the comparison between these equations for PMMA, Al₂O₃, and Zn. Results show that the prediction from Eq. (1-6) agrees satisfactorily with the experiments for all types of test particles.

Considering the environment of soot, it is indispensable to understand quantitatively the phenomenon for gas mixtures. Before analyzing the phenomenon for the gas mixture, it would be desirable also to know the characteristics of the phenomenon for each gas component, since the combustion gas mixture contains many gas species.
1.5 Scope and Objective of Present Work

The main objective of the present study is to investigate the influence of gas species on the thermophoretic velocity. Systematic investigations have been carried out in order to achieve the objective. For the first step, the characteristics of the thermophoresis for several pure gases, i.e., argon, nitrogen, and carbon dioxide are experimentally examined. Obtained experimental results are compared with theoretical predictions; a notable discrepancy is found for carbon dioxide, while the results for other two gases are consistent with predictions. Some attempts are made to fix the discrepancy: first by modifying constants and second by modifying two empirical coefficients in the theory. In the next step, an attempt is made to investigate dependence of the thermophoretic parameters on gas properties; the thermophoretic parameters for argon and nitrogen are reexamined, and compared with those of carbon dioxide, methane, and nitrous oxide. In
the final step, thermophoretic velocities are experimentally measured for several gas mixtures, i.e., argon-nitrogen, argon-carbon dioxide, and nitrogen-carbon dioxide. The tangential momentum accommodation coefficient is estimated from experimental result by assuming the thermal accommodation coefficient at unity, and it is compared with predictions calculated from values of composing pure gases by means of some methods given in references.

**Dissertation Outline**

In *Chapter 1*, brief introduction about the background and previous theoretical and experimental works of thermophoresis phenomenon is described. The motivations, scope and objective of the present study have also been addressed.

Experimental apparatus and particle sample used in the present study have been presented in *Chapter 2*. In the section of experimental apparatus, information about the configuration of experimental apparatus, the temperature control, and the relay control have been described. Information about the material, the diameter distribution, and the mean diameter of particles are reported in the section of particle sample.

The explanation about the experimental method is given in *Chapter 3*. Information about methods for measurement of the thermophoretic velocity, measurement of the temperature field, blowing particles, and experimental condition are presented.

Considering the environment of soot, it is indispensable to understand quantitatively the thermophoretic phenomenon for pure gases and gas mixtures. For the first step in this investigation, the characteristics of the thermophoresis for several gas
species have been experimentally examined in Chapter 4. Atmospheric gases chosen are pure gases of argon, nitrogen, and carbon dioxide. Obtained experimental results have been compared with theoretical predictions. Some attempts are made to fix the discrepancy for carbon dioxide.

It would be desirable to know physical relationship between the coefficients and the gas properties since the combustion gas mixture contains many gas species. An attempt is made to investigate dependence of the thermophoretic parameters on gas properties; the thermophoretic parameters are examined for argon and nitrogen, carbon dioxide, methane, and nitrous oxide. The residue values of all combinations of accommodation coefficients are calculated from the obtained experimental results. The tangential momentum accommodation coefficient is estimated by assuming the thermal accommodation coefficient to be unity. Relations between the tangential momentum accommodation coefficient and gas properties have been discussed and presented in Chapter 5.

The obtained experimental results in Chapter 5 show the coefficients are approximately the same for argon and nitrogen, while a significant difference is seen for carbon dioxide, methane and nitrous oxide. A problem arises here for the treatment of the tangential momentum accommodation coefficients for a mixture of pure gases having different values. It should better be confirmed even when the mixture is composed of pure gases that have the same coefficient value. Therefore, further investigation in Chapter 6 has been carried out to determine the tangential momentum accommodation coefficient that is estimated from the experimental results by assuming the thermal accommodation coefficient at unity. The estimated coefficient has been compared with
predictions calculated from values of composing pure gases by means of some methods given in the references to determine the appropriate method for evaluating the mixture’s coefficients. Gas mixtures chosen are argon-nitrogen, argon-carbon dioxide, and nitrogen-carbon dioxide.

Finally, the general conclusions of the current study and suggestion for further research have been addressed in Chapter 7.

References:


Chapter 1: Introduction


Chapter 1: Introduction


Chapter 2

Experimental Apparatus and Particle Sample

2 Experimental Apparatus and Particle Sample
2.1 Experimental Apparatus

2.1.1 Configuration of Experimental Apparatus

Experimental apparatus is divided into three main units: drop tower, drop unit and measuring unit. Table 2.1 shows the specification of equipment used in each unit. Each unit is explained as follows:

2.1.1.1 Drop Tower

Figures 2.1 and 2.2 show the overview and the schematic of the apparatus, which is included a drop tower, a measuring unit, and a damping cushion. The measuring unit is hung at the top of the drop tower by an electric magnet. The unit starts falling when the electric magnet is deactivated. The falling distance is 0.6 m, which corresponds to the duration time of the microgravity condition at 0.3 s. In this research, the drop tower with the height about 2 m is chosen based on consideration of the drop distance, the height of the drop unit, and height of the damping cushion.

The framework of the drop tower is W1 m × L0.7 m × H2 m and is manufactured from angle frames. The base of the tower is fixed to the floor by bolts for
preventing the movement of the tower during experiments. The electric magnet is attached to the center of the upper of the drop tower to hang the drop unit 1 m height from the floor. When the magnet is deactivated, the drop unit falls before it hits the damping cushion. The damping cushion is chosen from the commercial comforter and is placed inside the cardboard box with the size of W0.9 m × L0.6 m × H0.4 m.

Figure 2.3 shows a variation of the gravity level during an experiment. The gravity level is measured by the G-sensor attached on the measuring unit. The duration time of the free-fall is about 0.3 s as seen in the figure. When the electric magnet is deactivated at \( t = 0 \), the gravity level in z-axis changes from \(-1.0 \) G to \(+0.4 \) G once, possibly owing to the vibrational motion of the frame of the unit, and then decays as time passes until the unit reaches the cushion. The range between \( \pm0.1 \) G in gravity level is regarded as the microgravity condition in this work, the duration time of which is about 0.25 s.
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<td>DG85-31A-6C11</td>
</tr>
<tr>
<td>M</td>
<td>Seamless cylinder</td>
<td>Swagelok</td>
<td>304L-HDF2</td>
</tr>
<tr>
<td>N</td>
<td>PLC</td>
<td>OMRON</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>Switching power supply</td>
<td>TDK-Lambda</td>
<td>HK150A-24</td>
</tr>
</tbody>
</table>
Chapter 2: Experimental Apparatus and Particle Sample

Fig. 2.1  Overview of apparatus.

Fig. 2.2  Schematic of apparatus.
Chapter 2: Experimental Apparatus and Particle Sample

2.1.1.2 Drop Unit

Figure 2.4 shows the drop unit. The size of the unit is $W0.6 \, \text{m} \times L0.32 \, \text{m} \times H0.3 \, \text{m}$, which is constructed from the measurement unit, laser, electromagnetic valve, and steel plate. The angle frame is used to reduce the vibration during the drop. The steel wire and $\alpha$-gel cushion are assembled to the frame of the unit for reducing the impact at the beginning of the drop. The chain is attached to the steel plate, which is connected to the electric magnet at the drop tower to hang the drop unit. The unit is hanging 1.0 m height from the floor as seen in Fig. 2.2. When the electric magnet is deactivated, the unit drops and hits the cushion.
Fig. 2.4 Drop unit.

(a) Overview

(b) Top view

DR - 01

DR - 02

Fig. 2.4 Drop unit.
2.1.1.3 Measuring Unit

Figure 2.5 shows the schematic of the measuring unit. The unit includes the pressure vessel, the particle reservoir, the laser, and the high-speed camera. This unit is assembled on the aluminum base of the drop unit. Figure 2.6 shows the schematic of the pressure vessel, which is main part of the measuring unit.

(a) Overview of measurement unit

(b) Top view of measurement unit
Chapter 2: Experimental Apparatus and Particle Sample

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Fig. 2.5 Schematic of measuring unit.

(a) Pressure vessel

Figure 2.6 shows a cross-section of the pressure vessel. Placed in the vessel is a cylindrical thin chamber of 1.5 mm in height between two opposing aluminum plates of 10 mm × 80 mm × 80 mm. The upper plate, initially at the room temperature, is kept cool by moving it 15 mm away from the bottom plate while the latter is under the process of heating to raise its temperature, which is shown in Fig. 2.6(a). Keeping this arrangement, the vessel is evacuated by a pump. When the bottom plate reaches the appointed temperature, the upper plate is put in place where the distance between the plates is 1.5 mm as seen in Fig. 2.6 (b). Then, the vessel is filled with the test gas. The
temperature of the both plates is measured by two thermocouples, which are connected to the temperature control unit.

Fig. 2.6 Schematic of pressure vessel.

(a) Heating bottom plate

(b) During experiment

Fig. 2.6 Schematic of pressure vessel.
A sidewall of the vessel is attached with the mirror for capturing the image of particles via the high-speed camera. Figure 2.7 shows the coordinate system in the vessel. The origin is fixed at the center of the bottom plate. x, y, and z-axis’s represent the horizontal, the vertical, and the optical directions, respectively.

(a) Top view of pressure vessel

(b) Side view of pressure vessel

Fig. 2.7 Coordinate system in pressure vessel.
(b) Particles blowing

The method for particles blowing is explained by using Fig. 2.5. Both inlet and outlet valves are open when some particles are inserted to the particle reservoir. The pressure vessel is evacuated by the pump before it is refilled with the test gas to the target pressure. The compound pressure gauge is used to show the pressure. Later, the electromagnetic valves are closed and the small amount of the pressure is added to create a different pressure between the right side and the left side of the electromagnetic valve at the inlet. This pressure is indicated by the micrometer pressure gauge. Several preparatory experiments have been conducted to find the appropriate pressure for blowing the particles into the measurement field. The ball and needle valves are closed after setting the pressure. The particle reservoir includes the handle allowing the reservoir to open and close during the experiment. The reservoir is closed when the pressure vessel is evacuated and refilled with the test gas in order to prevent the particles in the reservoir from disappearing. The reservoir is opened after setting the blow pressure.

(c) Imaging unit

Those blown particles are illuminated by the laser beam introduced into the chamber to produce scattering lights, the image of which is recorded by a high-speed camera under the condition of the frame rate at 200 fps and of the shutter speed at 2 ms. The laser used in this research is chosen from the all-solid-state green laser. It is selected based on the high output power stability, ultra compact, good beam profile, and easy for operating. Since the distance between the plates is only 1.5 mm, the micro-lens is attached to the camera to enlarge the image of the measurement field.
2.2 Particle Sample

It is noted that the measurement accuracy of thermophoretic velocity is influenced by variation of the size and the shape of particles. In the present work, particles having a small variation of the size and the shape are chosen in order to increase the measurement accuracy. The thermophoretic velocity is derived from the balance between the thermophoretic force and the drag. The drag is estimated from the Stokes’ law. Even the shape of particles influences the drag on the particles, the Stokes’ approximation is applicable since Reynolds number is far less than one.

It is necessary to carefully choose the size of particles since the size influences on the Knudsen number, which one of the important parameter in the thermophoresis. The following conditions should be considered during the selection of the particle:

(1) Viscosity drag is verified.
(2) Physical characteristics are known.
(3) Particle size is in µm.
(4) Cohesion between particles is difficult to happen.
(5) Spherical particles.

Sample particles used in this research are PMMA (polymethylmethacrylate) sphere particle from Sekisui Plastics Co., Ltd. The density and the thermal conductivity of the particles are 1200 kg/m³ and 0.21 W/(m K), respectively. These particles are chosen since the size is quite uniform. Figures 2.8 and 2.9 show the SEM image and the probability density distribution of the particles diameter measured from the images, respectively. The mean diameter of these particles is 2.91 µm as shown in Table 2.2.
Chapter 2: Experimental Apparatus and Particle Sample

Fig. 2.8 SEM image of PMMA particles.

Fig. 2.9 Diameter distribution of particles.
Table 2.2  Diameters of PMMA particles

<table>
<thead>
<tr>
<th>Mean, µm</th>
<th>Standard deviation, µm</th>
<th>Confidence interval (95%) for the mean, µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.91</td>
<td>0.09</td>
<td>0.03</td>
</tr>
</tbody>
</table>
Experimental Method

3 Experimental Method

Experimental method is divided into four main sections: measurement of the thermophoretic velocity, measurement of the temperature field, method for blowing particles, and experimental condition. Each section is explained as follows:

3.1 Measurement of Thermophoretic Velocity

3.1.1 Reference Temperature

The thermophoresis is a phenomenon due to the momentum exchange from the collision between of gas molecules and the particle. The mean velocity of gas molecules increases as the temperature increases, such that the momentum of gas molecules increases accordingly. The measurement of the thermophoretic velocity should be taken at a fixed temperature since the velocity is dependent on not only the temperature gradient but also the temperature itself.

In this research, two experiments with different conditions are conducted: first is a pure gas and another is a gas mixture. For the former one, the thermophoretic velocity is measured at 313±2 K. Figure 3.1 (a) shows the position of the reference temperature; which is 1.0 mm from the hot surface. For the latter one, the measurement is performed at
338±10 K. The position of the reference temperature is determined at 1.2 mm from the hot surface as shown in Fig. 3.1 (b). It is noted that the position of the reference temperatures are different between these two experiments. The position is related to the measurement area. For the former experiment in carbon dioxide, most of particles drop quickly than the latter condition. The measurement could not be done since there is no particle in the measurement area when the measuring unit is dropped. Thus, the measurement area is changed, such that the position is changed accordingly, in order to easily capture the image of particles during the experiment.

(a) Pure gas
3.1.2 Measurement Software of Thermophoretic Velocity

The movement of particles is captured via a high-speed camera. The image is analyzed by using two-dimensional image measurement software. Table 3.1 shows the specification of the measurement software. This software analyzes the position for each selected particle from the image. The position is measured based on two-dimensional coordinate. The setting of the brightness and the contrast can be adjusted in order to make the image more clearly.
Table 3.1 Specification of measurement software.

<table>
<thead>
<tr>
<th>Name</th>
<th>Maker</th>
<th>Model</th>
<th>Hardware requirement</th>
</tr>
</thead>
<tbody>
<tr>
<td>A two-dimensional image measurement software</td>
<td>Library company</td>
<td>Move-tr / 2D 7.0</td>
<td>OS : Windows2000/XP/Vista</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>CPU : PentiumIII and above</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Memory : 256 MB and above</td>
</tr>
</tbody>
</table>

The thermophoretic velocity is calculated based on position of each particle provided via the measurement software. Figure 3.2 shows the position for several PMMA particles in surrounding gas of nitrogen. The result shows that the position of particle is proportional to the time. In order to obtain accurate results, the measurement of position should be larger than 10 scenes. The thermophoretic velocity for each particle is estimated by means of the least-squares method.

![Fig. 3.2 Movement of particles.](image-url)
Concerning the measurement from the software, there is a possibility that a particle is mistakenly selected. The position of each particle is confirmed from the graph in order to remove this error. Figure 3.3 shows the comparison of the movement between two particles. In the case of the particle noted as ‘A’, the position is constantly increased by the number of scene. On the other hand, in the case of the particle ‘B’, the notable change in the position is seen at scene number 24. The thermophoretic velocity is re-estimated after removing such error.

Fig. 3.3 Comparison of movement between particles.
3.1.3 Statistical Analysis of Measured Data

Experiments have been conducted repeatedly to accumulate data of the thermophoretic velocity. The satisfactory accuracy can be attained if the amount of data is sufficient for statistical treatment. Confidence interval is determined for indicating the range of expected mean value of the population.

For samples $X_1, X_2, \ldots, X_n$ from a normally distributed population with mean $m$ and variance $\sigma^2$, the probability density of the population is given as follows:

$$P\left\{ \frac{X - m}{\sigma/\sqrt{n}} \leq 1.96 \right\} = 0.95,$$

(3-1)

$$\overline{X} - 1.96 \frac{\sigma}{\sqrt{n}} \leq m \leq \overline{X} + 1.96 \frac{\sigma}{\sqrt{n}}$$

(3-2)

where $\overline{X}$ and $\sigma/\sqrt{n}$ are the sample mean and the standard error, respectively. Equation (3-2) expresses the 95% confidence interval of the mean for thermophoretic velocity. In this work, it is 95% confident that the mean value is in the confidence interval.

3.2 Measurement of Temperature Field

The temperature field is created by using the temperature control unit. The lower plate is heated via a plate heater while the upper plate is kept cold in order to perform the temperature gradient. When the bottom plate reaches its pointed temperature, the upper plate is put in the place where the distance between the plates is 1.5 mm. It should be kept in mind that the target temperature of cold plate should be higher than environmental temperature. The measuring unit is dropped when the temperature of upper plate becomes
Chapter 3: Experimental Method

the target temperature. Efficiency of the experiment is increased by preparing two upper plates; which are changing after every experiment.

The temperature field during the experiment is monitored by measuring temperatures at two points in the vessel: one is in the upper plate and the other in the bottom one. The temperature field is controlled based on these measured temperatures. Before making experiments, the preparatory measurement for the temperature field is conducted by inserting two more thermocouples suspended in the chamber at different heights. Measuring two points is sufficient because the linearity of the temperature field has already been confirmed by the Mach-Zehnder interferometry. The relation between these temperatures at monitoring points and the actual temperature field in the chamber is examined from this preparatory measurement. When conducting thermophoresis experiments, additional two thermocouples are removed; the temperature field is controlled based on the temperatures at those two monitoring points.

It is noted that the maximum temperature of the plate heater is 300ºC. The experiment should be conducted lower than this temperature. The heater plate is fixed 3 mm under the bottom plate, thus the temperature of bottom plate is considered to be the same as the temperature of the heater plate. Therefore the temperature of the bottom plate is controlled below than the maximum temperature of the heater plate.

Figure 3.4 shows a circuit diagram of temperature controller. Thermocouples I and II indicate the temperature of bottom and upper plates, respectively. Output I responses to the temperature setting from the thermocouple I. The output I is connected to the solid stay relay (SSR), which is used to control the temperature of bottom plate such that the temperature becomes constant when it reaches the target temperature.
Figure 3.5 shows the temperature variation for both plates during the experiment. Blue and red lines represent the temperature for bottom and upper plates, respectively. For example, the temperature of bottom plate is set at 210°C and the distance between plates is 1.5 mm. It is seen that the temperature of bottom plate becomes constant after the plate reaches its target temperature while the temperature of upper plate increases as time increases.

Fig. 3.4 Circuit diagram of temperature controller.
3.3 Method for Blowing Particles

The sequence controller is used for controlling each stage during the experiment such as recording the image of measurement field, falling of the drop unit, and blowing particles. Figure 3.6 shows the time chart during the experiment. The chart shows that after the high-speed camera is started, the electromagnet is deactivated to create the microgravity environment and particles movement in the measurement field are recorded. Figure 3.7 shows the ladder chart of programmable logic controller (PLC). Symbols T, B, M, and Q in the chart represent the timer, the button, auxiliary relay, and output relay, respectively. Q1, Q2, and Q3 are the electric magnet for holding the drop unit, the inlet electromagnetic valve, and the outlet electromagnetic valve, respectively.

The electromagnetic valves are open by pushing the button ⑤ on the PLC to evacuate and fill the vessel with the test gas. Both the electromagnetic valves are closed.
and the magnet for holding the drop unit is activated by pushing the button ④. When the temperature of the upper plate becomes the target temperature, the button ② is pushed such that both electromagnetic valves at the inlet and the outlet are simultaneously opened for 0.01 s. Some particles are blown into the chamber from a particle reservoir upstream the inlet. The magnet holding the measuring unit is deactivated 0.85 s after the closure of both valves. The disturbances caused by the blow are expected to cease within this period before the drop; since it is larger than both characteristic times for the flow field and the temperature field to reach the steady state; those characteristic times are expressed as \( t_f = (\delta/2)^2 / \nu \) and \( t_t = (\delta/2)^2 / \alpha \), respectively, where \( \delta \), \( \nu \), and \( \alpha \) are the distance between two plates, the kinematic viscosity, and the thermal diffusivity, respectively. The largest values of those for present conditions are estimated at 0.06 s and 0.05 s, respectively.

![Fig. 3.6 Time chart.](image-url)
The movement of particles in the microgravity environment has been confirmed by conducting the experiment without temperature gradient. Figures 3.8 and 3.9 show the movement of particles in the vertical and horizontal directions, respectively. It is seen that there is negligible effect of blowing and gravitational since the particles has almost no movement in the horizontal and the vertical directions.
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Fig. 3.8 Movement of particles in vertical direction.

Fig. 3.9 Movement of particles in horizontal direction.
3.4 Experimental Condition

3.4.1 Pure Gases

Experiments are conducted with the test gas of argon, nitrogen or carbon dioxide. The target temperature and the temperature gradient are 313 K and 10 K/mm, respectively. Various pressure conditions are chosen from 20 kPa to 100 kPa. The thermophoretic velocity for each particle is individually measured, and the mean value and its 95% confidence interval for each experimental condition are statistically obtained.

3.4.2 Gas Mixtures

The experimental configuration used in this work is the same as the pure gases except that a gas mixer is introduced. A mixture of two gases is created before it is inserted into the pressure vessel, in which particles are supplied to measure the thermophoretic velocity. The mixer is evacuated by a pump before two gases feed into it. Gas mixtures in various ratios are prepared by adjusting the partial pressure of each gas. The gases used in this work are argon, nitrogen, and carbon dioxide. The pressure, the reference temperature, and the temperature gradient of the surrounding gas are 70 kPa, 338 K, and 60 K/mm, respectively. It is noted that the temperature gradient in this work is increased, and the reference temperature is changed accordingly, from the previous condition such that the thermophoretic velocity increases and the measurement accuracy increases.
4.1 Introduction

Thermophoresis is the phenomenon that a small particle in a gas with a temperature gradient moves toward the lower temperature side. This phenomenon is supposed to influence practically on the movement of soot particles in exhaust gas from combustors, especially near cool walls around the hot gas flow, where the temperature gradient can become as large as the order of $10^2$ K/mm. For example, the measured size-distribution of particulate matter is influenced by the temperature of the transfer tube connecting the exhaust pipe of a diesel engine to the dilution tunnel ahead of the measuring device [1]. It is described in the Chapter 1 that there have been some direct measurements on the thermophoretic phenomenon of such an aggregate particle. The results of measurements on the thermophoretic velocity of soot particles suggested that the velocity is dependent not only on the macroscopic size of the soot particle but also on the aggregating condition [2, 3]. Suzuki et al. [3] have shown experimental results suggesting that the phenomenon is dominated by the size of primary spheres when the aggregation is coarse. Thus, the understanding of the phenomenon for a single sphere is indispensable before understanding for an aggregate.
The thermophoretic phenomenon of a spherical particle has been studied both theoretically and experimentally. The detail of these works is described in the Chapter 1. Experimental methods in the previous works are complex in practical implementation, and as a consequence, involve numerous errors. Among those, errors caused by buoyancy are the largest problem; in a field with a temperature gradient, buoyancy induces natural convection, which influences the movement of particles and disturbs the measurement. The velocity of such natural convection is usually comparable to the thermophoretic velocity, and cannot be measured directly. Toda et al. [4, 5] and Prodi et al. [6, 7] have conducted some experiments under microgravity conditions in order to avoid this problem. However, their reported data still seem to contain errors, possibly owing to limited trial numbers of experiments, so that those data are not sufficient to make quantitative comparison with theories [8].

Recently, our group has developed a device for conducting experiments repeatedly under a microgravity environment in a very short period time, i.e. 0.3 s, by means of the free-fall method, to accumulate data of the thermophoretic velocity. It has been confirmed that satisfactory accuracy can be attained if the amount of data is sufficient for statistical treatment [8]. By comparing the obtained experimental results with the existing theory [9], a notable difference is found for particles with high thermal conductivity [10]. The problem of the difference has been solved by reconsidering the boundary condition to improve the theory [11]. Hoshino et al. [12] has derived an improved theoretical solution of the thermophoretic velocity by applying the boundary condition proposed by Lockerby et al. [13], which includes the thermal stress slip and the higher order isothermal slip.
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Considering the soot environment, it is indispensable to understand quantitatively the phenomenon for a gas mixture. Before analyzing the influence of the composition of the gas mixture, it is indispensable to know the characteristics of the phenomenon for each gas component since the combustion gas mixture contains many gas species.

In this study, the characteristics of the thermophoresis for several gas species, i.e., argon, nitrogen, and carbon dioxide are experimentally examined. The first one is chosen as the reference, and the latter two are chosen as the major components of exhaust gas from combustors.

4.2 Theory for Thermophoretic Velocity

The theory adopted in this work is based on the works by Hoshino et al. [12] and by Chang and Keh [14]. The thermophoretic velocity is calculated from the balance between the thermophoretic and the drag forces, the equation of which is shown as below:

\[
V_T = \frac{2\mu C_f}{pT_{F_0}} \left[ C_s (k + C_r Kn) + C_h \left( C_M C_T Kn^2 + C_M Kn[k - 1]\right) \right] \left| \nabla T \right|, \tag{4-1}
\]

where \(\mu\), \(|\nabla T|\), \(p\), \(T_{F_0}\), \(C_s\), \(k\), \(Kn\), \(h\), \(C_M\), \(C_T\), \(C_s\), and \(C_H\) are the viscosity, the temperature gradient, the density of the surrounding gas, the reference temperature, the slip correction factor of the drag force for rarefied condition, the gas-to-particle thermal conductivity ratio, Knudsen number, the term of higher order isothermal slip, and constants for slip flow, temperature jump, thermal creep, and thermal stress slip, respectively. Here, the reference temperature \(T_{F_0}\) is defined as the supposed gas temperature at the center of the particle in the given temperature field without the existence of the particle, and Knudsen
number $Kn$ is the ratio of the mean-free-path $l$ to the particle radius. The mean-free-path is calculated from the following equation [15]:

$$l = \frac{\mu}{0.499P} \sqrt{\frac{\pi RT_{F,0}}{8}},$$  \hspace{1cm} (4-2)$$

where $P$ and $R$ are the pressure and the gas constant, respectively. The slip correction factor $C_f$ adopted in this work is Cunningham’s correction factor $C_{fc} = 1 + A Kn$, where $A = 1.257 + 0.4 \exp(-1.10/ Kn)$ [16]. The term $h$ of higher order isothermal slip is written as below:

$$h = \frac{9}{2\pi} PrKn^2(1 - \gamma^{-1}),$$  \hspace{1cm} (4-3)$$

where $Pr$ and $\gamma$ are Prandtl number and the specific heat ratio, respectively.

Equation (4-1) is basically identical to both of those proposed by Hoshino et al. [12] and Chang and Keh [14]. If constant values $C_S = 0.75$ and $C_H = 1.00$ [13] are applied, the equation becomes just the same as the one given by Hoshino et al. [12]. On the other hand, the equation becomes the same as the one given by Chang and Keh [14] by neglecting the term $h$, which is usually negligible in the range of $Kn \leq 0.1$, and applying Basset’s correction [17], $C_{fb} = (1 + 3C_M Kn)/(1 + 2C_M Kn)$, instead of the Cunningham’s correction for the slip correction factor $C_f$. The Basset’s correction is applicable in $Kn \leq 0.1$ while the Cunningham’s one in $Kn < 1$, and both are basically identical in those applicable ranges.

Two constants $C_M$ and $C_T$ are calculated as follows from two empirical coefficients, i.e., the tangential momentum accommodation coefficient $\alpha_m$ and the thermal accommodation coefficient $\alpha_t$:  
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\[ C_M = \frac{2 - \alpha_m}{\alpha_m}, \quad (4-4) \]

\[ C_T = \frac{15}{8} \left( \frac{2 - \alpha_t}{\alpha_t} \right), \quad (4-5) \]

These accommodation coefficients represent the magnitude of the momentum and energy exchange in the collision between gas molecules and the particle [15, 18], which are defined as below:

\[ \alpha_m = \frac{M_i - M_r}{M_i - M_s}, \quad (4-6) \]

\[ \alpha_t = \frac{E_i - E_r}{E_i - E_s}, \quad (4-7) \]

where \( M \) and \( E \) are average tangential components of the momentum and the energy flux normal to the particle surface, respectively, of gas molecules, and subscripts \( i, r, \) and \( s \) refer to incident molecules, molecules reflecting from the surface, and molecules leaving the surface in equilibrium with the surface, respectively. For example, \( \alpha = 1 \) corresponds to the situation that incident molecules achieve complete equilibrium with the particle surface before leaving, while \( \alpha = 0 \) corresponds to the situation of complete specular reflection.

In many cases, values of accommodation coefficients are simply assumed to be unity [10, 11]. There are some experimental measurements, e.g., Thomas and Lord [19] and Douglas [20], who measured accommodation coefficients on steel spheres and gas covered tungsten tube, respectively. Although those experiments report values different from unity under some conditions, yet there are no commonly accepted values.
4.3 Results

Figure 4.1 shows examples of the movement of particles during a free-fall in the surrounding gas of argon at 20 kPa. The measurement of the velocity should be taken at a fixed temperature since the thermophoretic velocity is dependent on not only the temperature gradient but also the temperature itself. The velocity of each particle is measured by tracing its movement while it travels within the range of the temperature between 313±2 K. It is seen from the figure that the velocity of each particle can be considered as constant in the range. The velocity is constant also in other two gases.

![Fig. 4.1 Movement of particles in surrounding gas of argon.](image)

Table 4.1 shows the statistical data for the tested pressure conditions with the argon gas. It is noted that the 95% confidence interval indicates not the range of data scattering but the range of expected mean value of the population. It is seen that the confidence interval is roughly estimated at around 0.01 mm/s to 0.02 mm/s. The ratio of the confidence interval to the mean value is only 3% for the pressure at 20 kPa. As the pressure increases, the velocity decreases, and as a consequence, the ratio tends to increase.
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Table 4.1  Statistical data of thermophoretic velocity in surrounding gas of argon.

<table>
<thead>
<tr>
<th>Pressure, $P$, kPa</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sampling number, $n$, –</td>
<td>39</td>
<td>35</td>
<td>37</td>
<td>33</td>
<td>47</td>
</tr>
<tr>
<td>Mean thermophoretic velocity, $\overline{V_T}$, mm/s</td>
<td>0.538</td>
<td>0.183</td>
<td>0.111</td>
<td>0.075</td>
<td>0.066</td>
</tr>
<tr>
<td>Standard deviation, $\sigma$, mm/s</td>
<td>0.053</td>
<td>0.044</td>
<td>0.026</td>
<td>0.030</td>
<td>0.036</td>
</tr>
<tr>
<td>Confidence interval (95%), mm/s</td>
<td>0.017</td>
<td>0.015</td>
<td>0.009</td>
<td>0.011</td>
<td>0.011</td>
</tr>
<tr>
<td>Ratio of confidence interval to mean thermophoretic velocity, –</td>
<td>0.03</td>
<td>0.08</td>
<td>0.08</td>
<td>0.15</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Figure 4.2 shows the thermophoretic velocity for each gas. The white rectangle (□), the black circle (●), and the white circle (○) represent experimental values for argon, nitrogen, and carbon dioxide, respectively. Error bars in the figure indicate the 95% confidence interval for the mean. The solid line is the prediction calculated by assuming constants to be identical to those in the previous work [11], namely, $C_M$, $C_T$, $C_S$, and $C_H$ to be 1.000, 1.875, 0.750, and 1.000, respectively.
Fig. 4.2 Pressure dependence of thermophoretic velocity in each gas. Error bars represent the confidence interval (95%) for the mean. Predictions are calculated under the assumption of $C_M = 1.000$, $C_T = 1.875$, $C_S = 0.750$, and $C_H = 1.000$. 
For argon and nitrogen, predictions are in good agreement with experiments throughout all the tested pressure conditions; the solid line runs through the range of the confidence interval at every tested pressure. On the other hand, for carbon dioxide, discrepancy is seen at low pressure conditions; the prediction is within the range of the confidence interval only when the pressure is at 100 kPa or 80 kPa. As the pressure decreases, discrepancy becomes notable; the experimental value at 20 kPa is 0.182 mm/s, which is 70% of the theory.

4.4 Discussion

It is quite interesting that the discrepancy is seen only for carbon dioxide. Also, previous work [11] has shown satisfactory accordance between the prediction and the experiment for the air. As noted earlier, the theory contains four constants, $C_M$, $C_T$, $C_S$, and $C_H$, and the former two of these are calculated from accommodation coefficients $\alpha_m$ and $\alpha_t$. These constants and coefficients have been determined empirically; different researchers have given different proposed values. Attempts should be made for finding suitable values to fit the prediction to these experimental results.

The first attempt is done by applying proposed values from references, which are shown in Table 4.2. The set of values noted as Case 1 is from the paper by Hoshino et al. [11], and those as Case 2 and 3 are both from the paper by Chang and Keh [14]. Figure 4.3 shows predictions together with experimental results. The vertical axis represents the reduced thermophoretic velocity, which is a dimensionless parameter defined as follows:

$$V_{TR} = \frac{V_T T_F}{\nu \nabla T},$$  \hspace{1cm} (4-8)
where $\nu$ is the kinematic viscosity of the gas. This parameter is often used when making comparison between different conditions. Lines labeled as Prediction 1, 2, and 3 are those predicted from Eq. (4-1) by applying values of Case 1, 2, and 3, respectively. For both argon and nitrogen, the best fit to experimental results is Prediction 1, and then Prediction 3 follows; the velocity predicted from Case 3 is higher than the experiment at $Kn = 0.25$ for both two gases, whereas that from Case 1 is within the error bar throughout all the measured range of $Kn$. For carbon dioxide, on the other hand, there is no line predicting satisfactorily the experimental result at $Kn = 0.15$, though Prediction 3 seems to be better than Prediction 1. Prediction 2 can be omitted from the discussion since it obviously disagrees with the experiment for all gases.

<table>
<thead>
<tr>
<th>Table 4.2</th>
<th>Constants for each case.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$C_M$, $C_T$, $C_S$, $C_H$, $\alpha_m$, $\alpha_t$, References</td>
</tr>
<tr>
<td>Case 1</td>
<td>1.000 1.875 0.750 1.000 (1.000) (1.000) [11]</td>
</tr>
<tr>
<td>Case 2</td>
<td>1.140 2.180 1.170 1.000 (0.935) (0.925) [14]</td>
</tr>
<tr>
<td>Case 3</td>
<td>1.140 2.180 1.170 3.000 (0.935) (0.925) [14]</td>
</tr>
</tbody>
</table>

*Values in bracket are from Eqs. (4-4) and (4-5) (not directly given in the reference).*
Fig. 4.3   Comparison between predictions and experiment in each gas. Error bars represent the confidence interval (95%) for the mean. Predictions are calculated using constants in Table 4.2.
Chapter 4: Effect of Gas Species

Provisionally, Case 1 is chosen as the best among three cases; Case 3 is not good enough for high $Kn$ for all tested gases in comparison to Case 1. Choosing Case 1 instead of Case 3 is also consistent with the previous work, in which experiments have been conducted in the air under the atmospheric pressure for particles of PMMA, alumina, and Zn [10]; as shown in Table 4.3, significant differences are seen between Case 3 and experimental results for alumina and Zn, whereas agreements are satisfactory for all particles with Case 1. In the theory, only the radius and the thermal conductivity are considered as particle-related parameters. It would be reasonable to assume that coefficients for CO$_2$ are different from those for other gases; Sharipov [21] has reviewed related papers and summarized accommodation coefficients for various gases, in which CO$_2$ values are notably different from Ar and N$_2$.

<table>
<thead>
<tr>
<th>Material</th>
<th>Prediction (Case 1)</th>
<th>Prediction (Case 3)</th>
<th>Experiment*</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMMA</td>
<td>0.167</td>
<td>0.210</td>
<td>0.172 ± 0.011</td>
</tr>
<tr>
<td>Al$_2$O$_3$</td>
<td>0.037</td>
<td>-0.035</td>
<td>0.036±0.008</td>
</tr>
<tr>
<td>Zn</td>
<td>0.049</td>
<td>-0.034</td>
<td>0.050±0.010</td>
</tr>
</tbody>
</table>

*From Suzuki et al. [10].

The second attempt is done by changing the accommodation coefficients from Case 1 by means of the least square fit. The residue $R$ representing the discrepancy between the experiment and the prediction is defined as follows:

$$R = \sum_{j=1}^{n} \left( V_{ij} - V_{pj} \right)^2, \quad (j = 1, \ldots, n)$$  \hspace{1cm} (4 - 9)
where $V_e$, $V_p$, and $n$ are thermophoretic velocities of the experiment and the prediction, and the number of the tested pressure conditions, respectively, and the subscript $j$ refers to each tested condition. The values for $C_S$ and $C_H$ are unchanged from the Case 1, and those for $C_M$ and $C_T$ are calculated from Eqs. (4-4) and (4-5), respectively, with accommodation coefficients $\alpha_m$ and $\alpha_t$ determined such that the residue $R$ becomes the lowest.

Table 4.4 shows the result of the least square fit for carbon dioxide. The best fit is attained with the coefficients noted as “without limitation” in the table. It should be noted that both values are clearly above unity, which is curious when interpreting the physical meaning of those values. Based on the kinetic theory, both those coefficients should lie between 0 and 1 [9, 15, 18]. The Cercignani-Lampis (CL) model provides a more physical description of the gas-surface interaction [21, 22], which allows $\alpha_m$ to vary between 0 and 2 while $\alpha_t$ remains between 0 and 1. In this CL model, $\alpha_m$ can exceed unity when the surface is rough. Considering this, additional work is done for the fitting by applying the limitation of $0 \leq \alpha_t \leq 1$ and $0 \leq \alpha_m \leq 2$, the result of which is shown as “with limitation” in the table. In this case, $R$ becomes the lowest when both the coefficients are below unity even $\alpha_m$ is allowed to exceed it.

Table 4.4 Estimated accommodation coefficients for carbon dioxide.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha_m$</th>
<th>$\alpha_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>without limitation</td>
<td>1.193</td>
<td>1.307</td>
</tr>
<tr>
<td>with limitation</td>
<td>0.841</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Figure 4.4 shows the comparison of predictions based on these values. It is seen that both predictions are in good agreement with the experiment within the range of the tested conditions. The difference between these two predictions is notable only under the condition of higher Knudsen number than the current work.

![Graph showing comparison of predictions for carbon dioxide.](image)

**Fig. 4.4** Comparison of predictions for carbon dioxide. Error bars represent the confidence interval (95%) for the mean. The solid and the broken lines represent predictions calculated with the coefficients from least-square-fit “without limitation” and “with limitation”, respectively.

In order to interpret the different coefficients for the carbon dioxide, authors have found two different points of view from literatures so far; one is the molecular weight, and the other the diameter of a molecule. The former is based on the work by Gronych et al. [23], who have conducted experiments to find out that the tangential momentum accommodation coefficient increases as the molecular weight of the gas decreases. This may be related to the influence of gas species. However, applying this effect for the explanation of the present results would not be appropriate; the molecular weight of argon is closer to carbon dioxide rather than nitrogen, while the thermophoretic characteristic of the argon is much closer to the nitrogen rather than the carbon dioxide.
Chapter 4: Effect of Gas Species

The latter is based on the work by Arya et al. [24], who have conducted a molecular simulation for a wall-slip phenomenon in rarefied gases. According to their result, the tangential momentum accommodation coefficient decreases as the collision diameter of the molecule increases. Among the gases used in this work, carbon dioxide has a notably larger collision diameter; based on the rigid sphere model, the diameter of argon, nitrogen, and carbon dioxide are 3.659 Å, 3.784 Å, and 4.643 Å, respectively [25]. This can explain qualitatively the results in this work if the coefficients “with limitation” are adopted for the carbon dioxide and those at unity are adopted for other two gases. The thermal accommodation coefficient “with limitation” is also consistent with the work by Winkler et al. [26], in which the coefficient $\alpha_t$ is reported to be approximately unity for water vapor, a polyatomic gas having a large collision diameter. The constants and coefficients, however, cannot be determined from the results in this work, since several combinations of those are possible to fit predictions to experimental results.
4.5 Conclusions

In this study, effects of gas species on the thermophoretic velocity are investigated, and following results are obtained:

1. Experimentally-obtained pressure dependence of the thermophoretic velocity for both argon and nitrogen are quantitatively in good agreement with the theory, while notable discrepancy is seen for carbon dioxide, if values for the constants $C_M$, $C_T$, $C_S$, and $C_H$ to be 1.000, 1.875, 0.750, and 1.000 are applied to the theory.

2. Values of those empirical constants proposed in some references were applied to the theory and compared with experiments; none of those has reduced the notable deviation for the carbon dioxide at $Kn = 0.15$.

3. The coefficients are calculated by means of the least square fit. The obtained values can be interpreted qualitatively by considering the effect of the molecular diameter on the tangential momentum accommodation coefficient. The proposed values of $C_M$, $C_T$, $C_S$, and $C_H$ for carbon dioxide are 1.378, 1.875, 0.750, and 1.000, although further discussion will be required for the determination of the values.
References:


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Chapter 4: Effect of Gas Species


Chapter 5

Dependence on Gas Properties

5.1 Introduction

It is described in Chapter 4 that several experiments have been conducted to investigate the influence of gas species on the thermophoretic velocity. The velocities have been measured for pure gases of argon, nitrogen, and carbon dioxide. The results of pure argon and nitrogen have agreed satisfactorily with the theory [1, 2] without any modification, while carbon dioxide has exhibited a noticeable deviation from the theory. It has been demonstrated in that chapter that this deviation can be corrected by modifying two empirical constants, i.e., the tangential momentum accommodation coefficient and the thermal accommodation coefficient, which both are involved in the theory and usually assumed to be unity [3 – 6].

Considering the environment of soot, one should know the accommodation coefficients of each gas species composing gas mixture. It would be desirable to specify physical relationship between the coefficients and the physical parameters for all gas species since the combustion gas contains many gas species. Some of these species are radical intermediates, e.g., OH, H, O, CH, etc. It is difficult for conducting the experiment using these gases since these gases are highly reactive.
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In this study, an attempt is made to investigate dependence of thermophoretic parameters on gas properties; the parameters for argon and nitrogen are reexamined and compared with those of carbon dioxide, methane, and nitrous oxide.

5.2 Calculation of Residue Value

The thermophoretic parameters are determined based on the value of the residue. The residue \( R \) representing the discrepancy between the experiment and the prediction, which is defined as follows:

\[
R = \frac{\sum_{j=1}^{n}(V_{ej} - V_{pj})^2}{n}, \quad (j = 1, \ldots, n) \tag{5-1}
\]

where \( V_{ej}, V_{pj}, \) and \( n \) are the thermophoretic velocity of the experiment, the prediction, and the number of the measured particles, respectively. The subscript \( j \) refers to each particle.

The theory [1, 2] of the thermophoretic velocity is derived from the balance between the thermophoretic force and the drag force, which is given below:

\[
V_T = \frac{3 \mu C_e}{2 \rho T f_0} \left[ \left( k + C_T K_n \right) + \frac{4}{3} \left( C_M C_T K_n^2 + C_M K_n[k-1] \right) \right] \frac{\nabla T}{(1 + 3 C_M K_n + \frac{9}{2\pi} Pr K_n^2 (1 - \gamma^{-1}))(1 + 2k + 2C_T K_n)}, \tag{5-2}
\]

where \( \mu, \nabla T, \rho, T_0, C_e, k, K_n, Pr, \gamma, C_M \) and \( C_T \) are the viscosity, the temperature gradient, the density of the gas, the reference temperature, the Cunningham’s correction factor, the gas-to-particle thermal conductivity ratio, Knudsen number, Prandtl number, the specific heat ratio, and constants for slip flow and temperature jump, respectively. \( C_M \) and \( C_T \) constants containing the tangential momentum accommodation coefficient \( \alpha_m \) and the thermal accommodation coefficient \( \alpha_t \), respectively, which are written as follows:
These accommodation coefficients represent the magnitude of the momentum and energy exchange in the collision between gas molecules and particles. Based on the kinetic theory, both coefficients should lie between 0 and 1 [7 – 9]. The Cercignani-Lampis (CL) model provides a more physical description of the gas-surface interaction [10, 11], which allows $\alpha_m$ to vary between 0 and 2 while $\alpha_t$ remains between 0 and 1. In this CL model, $\alpha_m$ can exceed unity when the surface is rough.

In this work, the values of residue $R$ are calculated for all combination of the coefficients.

5.3 Results

Figure 5.1 shows the reduced thermophoretic velocity for each gas species. The dimensionless reduced thermophoretic velocity is calculated as follows:

$$V_{TR} = \frac{V_T T_{F_0}}{\nu |\nabla T|},$$

where $\nu$ is the kinematic viscosity of the gas. The reduced velocity is often used when making comparison between different conditions. The black and the white symbols represent the obtained results from the previous work (Chapter 4) and this work, respectively. Lines represent predictions from different combinations of the coefficients, which will be explained later. Error bars in the figure indicate the 95% confidence interval for the mean. It is noted that the pressure, the reference temperature, and the
temperature gradient of those additional data from this work for argon, nitrogen, and carbon dioxide are 70 kPa, 338 K, and 60 K/mm, respectively. These results are added for increasing the number of experimental data such that the measurement accuracy increases. The residue values for all combination of the coefficients are calculated from the discrepancy between the experiment results and the prediction from Eq. (5-5).

![Graph](image1)

(a) Argon

![Graph](image2)

(b) Nitrogen
Chapter 5: Dependence on Gas Properties

\[ \alpha_m = 0.821, \quad \alpha_i = 1.000 \]

\[ \alpha_m = 0.600, \quad \alpha_i = 0.755 \]

\[ \alpha_m = 0.890, \quad \alpha_i = 1.000 \]

\[ \alpha_m = 0.600, \quad \alpha_i = 0.720 \]
Chapter 5: Dependence on Gas Properties

Figure 5.2 shows contours of residues for each gas species. Solid lines represent the contour of the residue. The residue values are calculated within the ranges of $0 \leq \alpha_t \leq 1$ and $0 \leq \alpha_m \leq 2$, which are the limitations of both coefficients. It is seen that the contours have positive orientations for all tested gases. The dotted line represents the best approximation of the thermal accommodation coefficient for each tangential momentum accommodation coefficient. The thermal accommodation coefficient is determined such that the residue $R$ becomes the lowest.

There is a problem for estimating the coefficients since the fitting gives several possible combinations. The same problem has been noted also in the Chapter 4. It is impossible to make rigorous calculation of the coefficients owing to the complexity of the equation of thermophoretic velocity or force [12]. Figure 5.3 shows the minimum value of the residue for each tangential momentum accommodation coefficient for argon. It is seen that there are several coefficients exhibit approximately the same values of the

![Fig. 5.1 Reduced thermophoretic velocity for each gas species.](image)

![Fig. 5.2 Contours of residues for each gas species.](image)
residues. A small increase up to only 4% is noticeable even the coefficient is reduced from 1.000 to 0.600. Comparisons of predictions between different combinations of the coefficients for each gas species are shown in Fig. 5.1. It is seen that for all gases, both predictions are in good agreement with the experiments within the range of the tested conditions.

In order to investigate the dependence of the coefficients on the gas properties, the tangential momentum accommodation coefficient is estimated by assuming the thermal accommodation coefficient to be unity. The assumption of the coefficient is made on the basis of results in the Chapter 4; the thermophoretic velocity can be predicted quantitatively by modifying the tangential momentum accommodation coefficient while the thermal accommodation coefficient is assumed at unity. The assumption is also consistent with the work by Winkler et al. [13], in which the coefficient is reported to be approximately unity for water vapor, a polyatomic gas having a large collision diameter.

The black arrow in Fig. 5.2 represents the estimated tangential momentum accommodation coefficient for each gas. The coefficients are approximately the same for argon and nitrogen, while a significant difference is seen in the coefficient between the other three gases.
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(a) Argon

(b) Nitrogen

(c) Carbon dioxide

Thermal accommodation coefficient, $\alpha_t$

Tangential momentum accommodation coefficient, $\alpha_{mv}$
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Fig. 5.2 Residue values for all combinations of coefficients for each gas species. The broken line represents best approximation of thermal accommodation coefficient for each tangential momentum accommodation coefficient.
5.4 Discussion

It is interesting that the tangential momentum accommodation coefficient differs among the tested gases. The different coefficients between gases can be interpreted by gas properties. Several researches have been done to investigate the dependence of the coefficient on gas properties. Arya et al. [14] have conducted a molecular simulation for a wall-slip phenomenon in rarefied gases flowing through micro- and nano-channels and have found that the coefficient decreases as the collision diameter of the molecule increases. Gronych et al. [15] have performed experiments to determine the coefficient by means of a viscosity vacuum gauge with a vibrating metal ribbon. The measurements show that the coefficient is dependent on the molecular weight; the coefficient of the lighter gas molecule is greater than that of heavier ones. Dadzie and Meolens [16] have proposed a new model of scattering kernels by considering more possible reflection types of molecules at a wall by employing a partial coefficient concept of various modes, i.e.,
translation, rotation, and vibration. The coefficient has been suggested to be influenced by the degree of freedom.

Table 5.1 shows the estimated tangential momentum accommodation coefficient and the gas properties for each gas species. It is seen that carbon dioxide has almost the same properties as nitrous oxide, while differences are found between other three gases.

<table>
<thead>
<tr>
<th>Gas</th>
<th>$\alpha_m$, -</th>
<th>Molecular weight, g/mol</th>
<th>Degree of freedom, -</th>
<th>Molecular diameter, Å</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argon</td>
<td>0.987</td>
<td>39.944</td>
<td>3</td>
<td>3.659</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>0.986</td>
<td>28.02</td>
<td>5</td>
<td>3.784</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>0.821</td>
<td>44.01</td>
<td>5</td>
<td>4.643</td>
</tr>
<tr>
<td>Methane</td>
<td>0.890</td>
<td>16.04</td>
<td>6</td>
<td>4.158</td>
</tr>
<tr>
<td>Nitrous oxide</td>
<td>0.836</td>
<td>44.02</td>
<td>5</td>
<td>4.662</td>
</tr>
</tbody>
</table>

Figure 5.4 shows the dependence of the tangential momentum accommodation coefficient on the molecular weight of gases. It is suggested that the coefficient decreases as the molecular weight increases [15]. Two disagreements are seen between the results in this work and the hypothesis given in the reference. In cases of methane, nitrogen, and argon, it is seen that the coefficient increases as molecular weight increases. In cases of carbon dioxide and nitrous oxide, the coefficients for both gases are smaller than the coefficient of argon even the molecular weights are approximately the same for those gases.
Figure 5.5 shows the dependence of the coefficient on the degree of freedom of gases. The result is inconsistent with the hypothesis that the coefficient is influenced by the degree of freedom [16]. It is seen that the results cannot be explained by a simple relationship between the coefficient and the degree of freedom. Two curious relationships are found from the results. First, the coefficients for argon and nitrogen are almost identical even both gases have different degree of freedom. Second, the coefficient for nitrogen is significantly greater than the coefficients for carbon dioxide and nitrous oxide, even nitrogen has the same degree of freedom with the other two gases.
Figure 5.6 shows the dependence of the coefficient on the molecular diameter of gas molecule. The results in this work can be explained qualitatively based on the effect of molecular diameter; the coefficient decreases as the diameter increases. These results are consistent with the work done by Arya et al. [14]. The value of coefficient is presumably dependent upon the interaction characteristic between gas molecules and the surface. The results infer that when small gas molecule collides with the surface, it loses the tangential momentum more than a large gas molecule.

![Diagram showing the relation between coefficient and molecular diameter.](image)

**Fig. 5.6** Relation between coefficient and molecular diameter.

There is a possibility that the coefficient is also dependent on the material of the solid; Thomas and Lord [17] have measured coefficients for polished and rough surfaces of steel spheres to find some influence of surface condition of the solid wall. However, this problem can be neglected in the present experiment since all experimental data are obtained from the same kind of particles.
5.5 Conclusions

In this study, the thermophoretic parameters of several gases are estimated from the experimental results, and following results are obtained:

1. The value of the residue $R$ is calculated from the discrepancy between the experiment results and the prediction for all combinations on the coefficients; several combinations of the coefficients are possible to fit the prediction with the experimental results.

2. The tangential momentum accommodation coefficient is estimated by assuming the thermal accommodation coefficient to be unity; the tangential momentum accommodation coefficient are approximately the same for argon and nitrogen, while a significant difference is seen for methane, nitrous oxide, and carbon dioxide.

3. A relation is shown between the tangential momentum accommodation coefficient and the molecular diameter; the coefficient decreases as the diameter increases.
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References:


Chapter 5: Dependence on Gas Properties


Chapter 6

**Effect of Molecular Fraction**

6.1 Introduction

As noted earlier, the theory of thermophoretic velocity contains two empirical constants, i.e., the tangential momentum accommodation coefficient and the thermal accommodation coefficient, both which are usually assumed to be unity [1 – 4]. In the Chapter 5, the accommodation coefficients have been estimated for pure gases of argon, nitrogen, carbon dioxide, methane, and nitrous oxide. The results show the coefficients are approximately the same for argon and nitrogen, while significant differences are seen for other three gases.

A problem arises here for the treatment of accommodation coefficients of a mixture of pure gases having different values, e.g., the mixture of nitrogen and carbon dioxide. It should better be confirmed even when the mixture is composed of pure gases having the same coefficient value, e.g. the mixture of argon and nitrogen, since there is no widely-accepted method so far for evaluating the coefficients of the mixture from the values of composing pure gases.

There are some theoretical and experimental researches dealing with one of those coefficients of gas mixtures, although none of those gives explicit discussion for both the coefficients together. Mikami et al. [5] have measured heat-transfer coefficients of a
sphere in hydrogen-nitrogen and helium-nitrogen mixtures by utilizing a thermistor, and have compared those results with an analytical solution based on the Maxwell’s model; good agreement has been shown by applying a linearly-dependent relation of the accommodation coefficient to the gas concentration. Wise et al. [6] have conducted experiments to measure the heat transfer with a conductivity cell in a gas mixture of atomic and molecular oxygens. The coefficient for pure atomic oxygen is estimated from extrapolation of experimentally-determined thermal accommodation coefficients of mixtures containing small amount of atomic oxygen. Ivchenko et al. [7, 8] have studied the slip phenomenon in the flow of a gas mixture to develop an analytical solution for planar transport problems. Bentz et al. [9] have performed experiments with a spinning rotor gauge to measure the slip flow constant, which is related to the tangential momentum accommodation coefficient, for helium-argon, helium-nitrogen, and helium-neon mixtures. Bentz et al.’s experimental results exhibit a nonlinear dependence on the mixture concentration; it seems to agree qualitatively well with the prediction from the Ivchenko et al.’s solution, though they have written of it “there are large differences”. Further investigation will be needed in order to determine the appropriate method for evaluating mixture’s coefficients.

In this study, thermophoretic parameters are experimentally investigated for several gas mixtures, i.e., argon-nitrogen, argon-carbon dioxide, and nitrogen-carbon dioxide. The dependence of the tangential momentum accommodation coefficient on the concentration of the mixture is examined from the measured thermophoretic velocity.
6.2 Factors Affected by Gas Mixing

The problem for a gas mixture is the treatment of two empirical parameters, i.e., tangential momentum- and thermal-accommodation coefficients, which characterize the interaction between the solid surface of the particle and surrounding gas molecules. In the gas mixture, a diffusiophoretic force will be exerted on a particle in addition to the thermophoretic force since the temperature gradient causes a concentration gradient. The thermodiffusiophoretic velocity for a gas mixture is shown as below:

\[
V_{TD} = \frac{3\mu C_c}{2\rho T_{F0}} \left[ (k + C_T Kn) + \frac{4}{3} \left( C_w C_T Kn^2 + C_M Kn[k - 1] \right) \right] \left[ \nabla T \right] + C_c' D_{12} \left[ \nabla x_1 \right], \quad (6 - 1)
\]

where \( \mu, \left[ \nabla T \right], \rho, T_{F0}, C_c, k, Kn, Pr, \gamma, C_M, C_T, c', D_{12}, \) and \( \left[ \nabla x_1 \right] \) are the viscosity, the temperature gradient, the density of the gas, the reference temperature, the Cunningham’s correction factor, the gas-to-particle thermal conductivity ratio, Knudsen number, Prandtl number, the specific heat ratio, the constant for slip flow, the constant for temperature jump, the diffusion slip factor, the coefficient of molecular diffusion, and the concentration gradient, respectively. Here, the reference temperature \( T_{F0} \) is defined as the supposed gas temperature at the center of the particle in the given temperature field without the existence of the particle. The first term in the right hand side of eq. (6 - 1) is the thermophoretic term, which is the same as in the previous Chapters 4 and 5. The second term is the diffusiophoretic term [10], which is detail discussed in Appendix B. The density \( \rho \), the viscosity \( \mu \), and the thermal conductivity of gas \( k_f \) for a binary gas mixture are calculated as follows [11]:

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Chapter 6: Effect of Molecular Fraction

\[ \rho = \frac{P_1}{R_1 \cdot T} + \frac{P_2}{R_2 \cdot T}, \quad (6 - 2) \]

\[ \mu = \sum_{j}^{n} \frac{\mu_j}{1 + \sum_{j=1}^{n} \phi_{jn} \left( x_n / x_j \right)}, \quad (6 - 3) \]

\[ k_j = \sum_{j}^{n} \frac{k_{jj}}{1 + \sum_{j=1}^{n} \phi_{jn} \left( x_n / x_j \right)}, \quad (6 - 4) \]

Here,

\[ \phi_{jn} = \left( \frac{1 + \left( \frac{\mu_j}{\mu_n} \right) \left( \frac{M_n}{M_j} \right)^{1/2}}{2 \sqrt{2} \left( 1 + \left( \frac{M_j}{M_n} \right) \right)^{1/2}} \right)^2, \quad (6 - 5) \]

\[ \phi_{jn} = \frac{1}{4} \left( 1 + \left[ \frac{\mu_j}{\mu_n} \left( \frac{M_n}{M_j} \right) \right]^{1/4} \left( \frac{1 + \left( \frac{C_j}{T} \right)}{1 + \left( \frac{C_n}{T} \right)} \right) \left( \frac{1 + \left( \frac{C_{jn}}{T} \right)}{1 + \left( \frac{C_j}{T} \right)} \right)^2 \right)^{1/2}, \quad (6 - 6) \]

\[ C_{jn} = \left( C_j \cdot C_n \right)^{0.5} \quad (6 - 7) \]

where \( R, C, M, \) and \( x \) are the gas constant, Sutherland constant, the molecular weight and the molar fraction, respectively. Subscripts \( j \) and \( n \) refer to the number of each gas species and the total number, respectively. Parameters \( c', D_{12}, \) and \( |\nabla x_j| \) are calculated by complex equations given by Kihara [12] and Ivchenko et al. [8].

Constants \( C_M \) and \( C_T \) in eq. (6 - 1) are the ones containing the tangential momentum accommodation coefficient \( \alpha_m \) and the thermal accommodation coefficient \( \alpha_t \), respectively, which are written as follows:

\[ C_M = \frac{2 - \alpha_m}{\alpha_m}, \quad (6 - 8) \]
In order to apply this same formula to a gas mixture, those coefficients of the mixture should be determined from values of each pure gas components.

In this work, only the tangential momentum accommodation coefficient is derived from experimental results, and the other one, the thermal accommodation coefficient, is assumed to be unity on the basis of results in the Chapter 5, in which all the thermal accommodation coefficients have been estimated at unity for the same pure gases adopted in this work.

There are three different methods for the calculation of the tangential momentum accommodation coefficient of a mixture, which are explained below:

i. Energy-balance-based method

The first one is given by Mikami et al. [5], who derived the tangential momentum accommodation coefficient for a gas mixture from the energy balance as below:

\[
C_r = \frac{15}{8} \left( \frac{2 - \alpha_r}{\alpha_r} \right) \tag{6-9}
\]

\[
\alpha_{m,mix} = \frac{\sum_{j=1}^{2} x_j \alpha_{m,j}}{\sum_{j=1}^{2} \frac{x_j}{\sqrt{M_j}}} \tag{6-10}
\]

where \( x \) and \( M \) are the molar fraction and the molecular weight, respectively, and subscript \( j \) refers to the number of each gas species.

ii. Mass-fraction-based method

The second one is found in the ANSYS FLUENT theory guide [13], in which both the accommodation coefficients of a mixture are calculated simply by taking the mass-fraction weighted average of each gas species as follows:
\[ \alpha_{\text{mix}} = \sum_{j=1}^{2} y_j \alpha_j, \]  

(6–11)

where \( y_j \) and \( \alpha \) are the mass fraction of each gas species \( j \) and each accommodation coefficient, respectively.

iii. Intermolecular-interaction-based method

The last one can be derived from equations given by Ivchenko et al. [7, 8]. The constant \( C_M \) for the slip-flow of a gas mixture was derived using the first-order Chapman-Enskog approximation and the Maxwell method as below:

\[ C_M = \frac{5}{16} \pi \frac{P \sqrt{x_1 M_1 + x_2 M_2}}{\mu} \frac{(2 - \alpha_{1\text{m}}) x_1 b_1 + (2 - \alpha_{2\text{m}}) x_2 b_2}{\alpha_{1\text{m}} x_1 \sqrt{M_1} + \alpha_{2\text{m}} x_2 \sqrt{M_2}}, \]  

(6–12)

where \( P \) and \( b \) are the pressure and the transport coefficient [7, 8] of viscosity corresponding to arbitrary models of the intermolecular interaction, respectively, and subscripts 1 and 2 refer to components of the binary gas mixture. Here, it is noted that the literature adopts the following equation instead of eq. (6 - 8):

\[ C_M = \frac{5}{16} \pi \frac{(2 - \alpha_m)}{\alpha_m}. \]  

(6–13)

This equation is basically identical to the eq. (6 - 8): the additional coefficient of \( 5\pi/16 \) to the eq. (6 - 8) makes the resultant value only 2% less. By equating those two eqs. (6 - 12) and (6 - 13), the tangential momentum accommodation coefficient of the mixture can be formulated as a function of the coefficients of two pure gases as follows:

\[ \alpha_{m,\text{mix}} = 2 \left( 1 + \frac{P \sqrt{x_1 M_1 + x_2 M_2}}{\mu} \frac{(2 - \alpha_{1\text{m}}) x_1 b_1 + (2 - \alpha_{2\text{m}}) x_2 b_2}{\alpha_{1\text{m}} x_1 \sqrt{M_1} + \alpha_{2\text{m}} x_2 \sqrt{M_2}} \right)^{-1}. \]  

(6–14)
6.3 Results

Figure 6.1 shows measured velocities for argon-nitrogen, argon-carbon dioxide, and nitrogen-carbon dioxide mixtures. Lines represent predictions by above-mentioned methods, which will be explained later. Error bars in the figure indicate the 95% confidence interval for the mean.

Nitrogen exhibits the highest thermophoretic velocity, followed by argon and carbon dioxide; the thermophoretic velocities for pure nitrogen, argon, and carbon dioxide are 0.880 mm/s, 0.673 mm/s, and 0.273 mm/s, respectively. For the mixture of argon-nitrogen, the thermophoretic velocity increases as the concentration of nitrogen increases. For mixtures of argon-carbon dioxide and nitrogen-carbon dioxide, the thermophoretic velocity decreases as the concentration of carbon dioxide increases.

Figure 6.2 shows tangential momentum accommodation coefficients of pure gases of argon, nitrogen, and carbon dioxide. The accommodation coefficient $\alpha_m$ for each experimental datum is calculated from eqs. (6 - 1), (6 - 8), and (6 - 9) by assuming $\alpha_t = 1$, and the mean value and its 95% confidence interval are statistically obtained for each experimental condition. White and black symbols represent values from the previous (Chapter 4) and this works, respectively. Solid lines indicate mean values among all pressure conditions. Coefficients from this work seem to be consistent with those from the previous.
Fig. 6.1 Thermophoretic velocity for each gas mixture. Error bars represent the confidence interval (95%) for the mean.
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Fig. 6.2 Tangential momentum accommodation coefficient for each pure gas. Error bars represent the confidence interval (95%) for the mean.

(a) Argon

(b) Nitrogen

(c) Carbon dioxide
Table 6.1 shows mean values of the coefficient from all the pressure conditions. Coefficients for argon and nitrogen are almost the same: 0.987 and 0.986, respectively. That for carbon dioxide exhibits a notably low value at 0.821.

Table 6.1  Tangential momentum accommodation coefficient $\alpha_m$ for each pure gas.

<table>
<thead>
<tr>
<th>Gas</th>
<th>$\alpha_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argon</td>
<td>0.987</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>0.986</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>0.821</td>
</tr>
</tbody>
</table>

Table 6.2 shows the tangential momentum accommodation coefficient calculated from experimental results. Error ranges indicate the 95% confidence interval for the mean. For argon-carbon dioxide and nitrogen-carbon dioxide, it is seen that the coefficient is dependent on the gas concentration. For the mixture of argon-nitrogen, the slightly increase up to 1% is noticeable in the mean value. However, it is difficult to make a statistically significant since the mean values are within the error range.

Table 6.2  Estimated tangential momentum accommodation coefficient $\alpha_m$ for each gas mixture.

<table>
<thead>
<tr>
<th>Ratio Ar:N$_2$</th>
<th>Ar:CO$_2$</th>
<th>N$_2$:CO$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00:0.00</td>
<td>0.987 ± 0.028</td>
<td>0.987 ± 0.028</td>
</tr>
<tr>
<td>0.75:0.25</td>
<td>0.989 ± 0.027</td>
<td>0.991 ± 0.025</td>
</tr>
<tr>
<td>0.50:0.50</td>
<td>0.993 ± 0.031</td>
<td>0.949 ± 0.029</td>
</tr>
<tr>
<td>0.25:0.75</td>
<td>0.992 ± 0.026</td>
<td>0.877 ± 0.022</td>
</tr>
<tr>
<td>0.00:1.00</td>
<td>0.986 ± 0.026</td>
<td>0.821 ± 0.031</td>
</tr>
</tbody>
</table>
Figure 6.3 shows comparisons of the tangential momentum accommodation coefficient of experiments and predictions from eqs. (6 - 10), (6 - 11), and (6 - 14). In cases of gas mixtures of argon-nitrogen and argon-carbon dioxide, predictions by the energy-balance and the mass-fraction methods are almost identical. On the other hand, in the case of nitrogen-carbon dioxide, predictions by the energy-balance and the intermolecular-interaction methods give close results while the mass-fraction method is distinguishable from others. Although there is slightly difference between methods, all predictions agree satisfactorily with the experimental result for argon-nitrogen. For argon-carbon dioxide, disagreements are seen between the experimental result and predictions by the energy-balance and the mass-fraction methods. For nitrogen-carbon dioxide, slightly disagreement is noticeable between the mass-fraction method and the experimental result at 25% carbon dioxide. The intermolecular-interaction method seems to be the best for all those mixture conditions.

Comparisons of thermophoretic velocities between predictions and experiments for gas mixtures are shown in Fig. 6.1. Although the difference is not so remarkable, the intermolecular-interaction method seems to be the best among compared three methods; the energy-balance and the mass-fraction methods underestimate the velocity for argon-carbon dioxide.
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Fig. 6.3 Comparison of tangential momentum accommodation coefficient for each gas mixture.
6.4 Discussion

A problem in the determination of coefficients of a gas mixture is the uncertainty of those values of composing pure gases. Different researchers have proposed different values for a same pure gas. Table 6.3 shows comparisons of coefficients between several references for the concerned pure gases; the values obtained in this work are also shown. These differences in the table infer the possibility that the accommodation coefficient is dependent on the adopted method and/or the material of the solid. The surface condition of the solid wall may also have some influence: Thomas and Lord [14] have measured accommodation coefficients for polished and rough surfaces of steel spheres to find some difference as shown in the table.

This problem should be considered when making quantitative comparison between values from different research works. With regard to the current and its preceding works, all experimental data are obtained from the same method and the same kind of particles. Thus, the comparison given in this work will not be concerned directly by the problem.
Table 6.3 Comparison of coefficients for pure gases.

<table>
<thead>
<tr>
<th>Tested gas</th>
<th>$\alpha_m$</th>
<th>$\alpha_t$</th>
<th>Solid material</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argon</td>
<td>0.931</td>
<td>1.102</td>
<td>Steel spheres (polished surface)</td>
<td>[14]</td>
</tr>
<tr>
<td></td>
<td>1.049</td>
<td>1.161</td>
<td>Steel spheres (rough surface)</td>
<td>[14]</td>
</tr>
<tr>
<td></td>
<td>0.916</td>
<td>1.000</td>
<td>Glass</td>
<td>[15]</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.900</td>
<td>Paraffin</td>
<td>[3]</td>
</tr>
<tr>
<td></td>
<td>0.987</td>
<td>1.000</td>
<td>PMMA</td>
<td>This work</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>1.000</td>
<td>0.680</td>
<td>Tungsten</td>
<td>[16]</td>
</tr>
<tr>
<td></td>
<td>0.911</td>
<td>1.000</td>
<td>Glass</td>
<td>[15]</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.900</td>
<td>Paraffin</td>
<td>[3]</td>
</tr>
<tr>
<td></td>
<td>0.986</td>
<td>1.000</td>
<td>PMMA</td>
<td>This work</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>1.000</td>
<td>0.720</td>
<td>Tungsten</td>
<td>[16]</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.450</td>
<td>Glass</td>
<td>[17]</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.450</td>
<td>Polystyrene latex (PSL) sphere</td>
<td>[17]</td>
</tr>
<tr>
<td></td>
<td>0.993</td>
<td>1.000</td>
<td>Glass</td>
<td>[15]</td>
</tr>
<tr>
<td></td>
<td>0.821</td>
<td>1.000</td>
<td>PMMA</td>
<td>This work</td>
</tr>
</tbody>
</table>

Another problem is the assumption that the thermal accommodation coefficient is always at unity regardless of the mixing condition. This does not concern when applying energy-balance- and mass-fraction- based methods, since both these methods do not alter the coefficient of the mixture when both the pure gases have the same value at unity. The intermolecular-interaction-based method, on the other hand, may change the value of the thermal accommodation coefficient depending on the concentration as is the case of the
tangential momentum accommodation coefficient. So far, there is no reference that shows the way to calculate the thermal accommodation coefficient on the intermolecular interaction basis. Although there are such vulnerable points in the treatment of coefficients, the current work would be informative for a practical use in predicting the thermophoretic velocity for gas mixtures. Experimental results are satisfactorily predicted when the thermal accommodation coefficient is assumed to be unity and the other coefficient is calculated from pure gas values by means of the intermolecular-interaction-based method.
6.5 Conclusions

In this study, the thermophoretic velocity of gas mixtures are experimentally measured, and following results are obtained:

1. The thermophoretic velocity depends on the concentration of the gas mixture. For an argon-nitrogen mixture, the velocity increases as the concentration of nitrogen increases; for argon-carbon dioxide and nitrogen-carbon dioxide mixtures, the velocity decreases as the concentration of carbon dioxide increases.

2. The tangential momentum accommodation coefficient of a gas mixture is estimated from experimental results to clarify its dependence on the concentration by assuming the thermal accommodation coefficient at unity. The estimated coefficient is compared with predictions from three methods; it is found that the intermolecular-interaction-based method gives the best prediction among those three methods for all mixture conditions.

3. It is shown that the thermophoretic velocity under a binary gas mixture condition is practically predicted from the intermolecular-interaction-based method.
References:


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Chapter 7

Conclusions and Future Work

7.1 Conclusions

In the present work, influences of gas species on the thermophoretic velocity of PMMA particles are investigated. The main conclusions of this study are summarized as follows.

In the present study, effects of gas species on the thermophoretic velocity are investigated. The characteristics of the thermophoresis for argon, nitrogen, and carbon dioxide are experimentally examined. The obtained-experimental results are compared with the theory. It is seen that when values for the constants $C_M$, $C_T$, $C_S$, and $C_H$ to be 1.000, 1.875, 0.750, and 1.000 are applied to the theory, results for both argon and nitrogen are quantitatively in good agreement with the theory, while notable discrepancy is seen for carbon dioxide. The discrepancy for carbon dioxide is corrected by changing values of coefficients, which are calculated by means of the least square fit. It is found the proposed values of $C_M$, $C_T$, $C_S$, and $C_H$ for carbon dioxide are 1.378, 1.875, 0.750, and 1.000.

The attempt is made to investigate dependence of thermophoretic parameters on gas properties. The thermophoretic parameters for argon and nitrogen are reexamined, and compared with those of carbon dioxide, methane, and nitrous oxide. The value of
residue $R$ is calculated for all combinations of coefficients. It is found that several combinations of the coefficients are possible to fit the prediction with the experimental results. The physical relationships between the tangential momentum accommodation coefficient and the gas properties are clarified by assuming the thermal accommodation coefficient to be unity. A relation is shown between the tangential momentum accommodation coefficient and the molecular diameter; the coefficient decreases as the diameter increases.

The thermophoretic velocity is experimentally measured for several gas mixtures, i.e., argon-nitrogen, argon-carbon dioxide, and nitrogen-carbon dioxide. It is found that the thermophoretic velocity depends on the concentration of the gas mixture. In the case of an argon-nitrogen mixture, the velocity increases as the concentration of nitrogen increases. On the other hand, in cases of argon-carbon dioxide and nitrogen-carbon dioxide mixtures, the velocity decreases as the concentration of carbon dioxide increases. The tangential momentum accommodation coefficient of a gas mixture is estimated from experimental results. The dependence of the tangential momentum accommodation on the concentration is clarified by assuming the thermal accommodation coefficient at unity. The comparison is done between the estimated coefficient and predictions from three methods. It is found that the intermolecular-interaction-based method gives the best prediction among those three methods for all mixture conditions. It is shown that the thermophoretic velocity under a binary gas mixture condition is practically predicted from the intermolecular-interaction-based method.
7.2 Recommendations for Future Work

Influences of composition of gas mixture on thermophoretic velocity have been investigated in the present study. However, due to some limitations, other influences could not be examined in this study. Therefore, several suggestions for future works related to the influences on the thermophoresis phenomenon are given as follows:

1. The water vapor is the clue of this work. However, it is excluded owing to technical reasons on conducting experiment. For this purpose, the apparatus need to be modified so that experiments can be conducted in such condition.

2. The influence of thermal conductivity of particle would be an important topic to be explored. Experiments for different type of particles should be considered to investigate this influence.

3. Considering the combustion environment, the thermophoresis phenomenon is supposed to influence on the movement of soot particles. The soot particle is generally an aggregate particle. The understanding of the phenomenon for an aggregate would be interesting topic in the future work.
Appendix A

Thermophoretic Velocity

The thermophoresis is caused by the momentum exchange between gas molecules and the particle. The primary mechanism of this exchange is that gas molecules colliding from the higher temperature side transfer negative momentums to the particles, the overall amount of which exceeds those of positive momentums transferred by gas molecules colliding from the lower temperature side. As the result of this momentum exchange, the particle and the surrounding gas move in mutually opposite directions. The flow of the surrounding gas is referred to as the superficial slip flow.

A-1 Calculation

A-1-1 Assumptions

For determining the governing equations and boundary conditions, the following assumptions are made:

1. A spherical particle is placed into the gas having a constant temperature gradient and fixed in its position.
2. The flow is steady and symmetrical with respect to the z axis, which corresponds to the direction of the temperature gradient.
3. The gas is incompressible.
4. Stoke’s approximation is applicable.

A-1-2 Coordinate System

Spherical coordinates are adopted in this calculation. The relation between the spherical coordinates and a Cartesian coordinates is shown in Fig. A-1 and Eq. (A-1).

\[ x = r \sin \theta \cos \phi, \]
\[ y = r \sin \theta \sin \phi, \]
\[ z = r \cos \theta. \]  \hspace{1cm} (A-1)

The center of the spherical particle is located at the origin of the coordinates.

A-1-3 Governing Equations

The incompressible steady Navier-Stokes equation with Stokes’ approximation is shown as follows:

\[ \mu \nabla^2 \vec{V} = \nabla p, \]  \hspace{1cm} (A-2)
where $\mu$, $\bar{V}$, and $p$ are the viscosity, the stream vector, and the pressure, respectively.

Equations of continuity, energy for the gas surrounding the particle, and heat conduction in the spherical particle are shown below:

\[ \nabla \cdot \bar{V} = 0, \quad (A-3) \]

\[ \nabla^2 T_F = \frac{1}{\alpha} \left( \bar{V} \cdot \nabla \right) T_F, \quad (A-4) \]

\[ \nabla^2 T_S = 0, \quad (A-5) \]

where $T_F$ and $T_S$ are temperatures of the gas and the particle, respectively.

\section*{A-1-4 Boundary Conditions}

The matching condition for the thermal flux at the surface is shown below:

\[ k_S \frac{\partial T_S}{\partial r} = k_F \frac{\partial T_F}{\partial r}, \quad r = a \quad (A-6) \]

where $k_S$ and $k_F$ are thermal conductivities of the particle and the gas, respectively, $a$ is the radius of the particle.

The temperature jump condition at the surface is given as follows:

\[ T_F - T_S = C_T l \frac{\partial T_F}{\partial r}, \quad r = a \quad (A-7) \]

where $l$ is the mean free path and $C_T$ is a constant for the temperature jump.

The superficial slip condition on the surface of the spherical particle is shown as below:

\[ V_\theta = C_M \left[ r \frac{\partial}{\partial r} \left( \frac{V_\theta}{r} \right) + \frac{1}{r} \frac{\partial V_r}{\partial \theta} \right] + \frac{3}{4} \frac{\mu}{\rho_{F0}} \frac{1}{a} \frac{\partial T_F}{\partial \theta} - C_M l \frac{\mu}{\rho_{F0}} \left( \frac{1}{r} \frac{\partial^2}{\partial r \partial \theta} - \frac{1}{r^2} \frac{\partial}{\partial \theta} \right) T_F \]

\[ - \frac{9}{4\pi} \frac{Pr(\gamma - 1)l^2}{\gamma} \frac{\partial \omega}{\partial r}, \quad r = a \quad (A-8) \]
where $C_M$ is the constant for slip flow, $V_\theta$ and $V_r$ are $\theta$ and $r$ elements of the stream vector, respectively, $\rho$ is the density of the gas, $T_{F0}$ is the reference temperature, $\omega$ is the element of vorticity in the $\phi$ direction, $Pr$ is the Prandtl number, and $\gamma$ is the ratio of specific heat.

In addition to the superficial slip condition, following boundary conditions are commonly set. Firstly, $V_r$ should be zero at the surface.

$$V_r = 0, \quad r = a \quad (A-9)$$

The faraway conditions are given as follows:

$$V_r = 0, V_\theta = 0, T_r = |\nabla T| r \cos \theta + T_{F0}, \quad r \to \infty, \quad (A-10)$$

where $|\nabla T|$ is the temperature gradient given to the domain. The axisymmetric conditions around $z$ axis are shown below:

$$\frac{\partial V_r}{\partial \theta} = 0, V_\theta = 0, \frac{\partial T_r}{\partial \theta} = 0, \frac{\partial T_\theta}{\partial \theta} = 0, \quad \theta = 0, \pi. \quad (A-11)$$

### A-1-5 Derivation of Thermophoretic Force

The thermophoretic force $F_T$ is derived by integrating the superficial stress as follows:

$$F_T = 2\pi a^2 \int_0^\pi \left( \tau_r \cos \theta - \tau_\theta \sin \theta - p \cos \theta \right) \sin \theta \, d\theta, \quad (A-12)$$

$$\tau_r = 2\mu \frac{\partial V_r}{\partial r}, \quad (A-12)$$

$$\tau_\theta = \mu \left( r \frac{\partial}{\partial r} \left( \frac{V_\theta}{r} \right) + \frac{1}{r} \frac{\partial V_r}{\partial \theta} \right),$$

where $\tau_r$ and $\tau_\theta$ are the normal and the tangential components of viscous stress, respectively. The reference point of the pressure is settled at $r = a, \theta = 0$. 

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A-1-5 Derivation of Thermophoretic Velocity

The thermophoretic velocity is derived from the balance between the thermophoretic force and the drag force. The equation is shown as follows:

\[ V_T = \frac{3\mu C_c}{2\rho T_F} \left[ \frac{(k + C_f Kn) + \frac{4}{3} \left( C_M C_f Kn^2 + C_M Kn[k - 1] \right)}{1 + 3C_M Kn + \frac{9}{2\pi} PrKn^2 (1 - \gamma^{-1}) \left( 1 + 2k + 2C_f Kn \right)} \right] \nabla T. \]  

(A–13)

Here, \( C_c = 1 + A Kn \), where \( A = 1.257 + 0.4 \exp(-1.10 / Kn) \).
Appendix B: Thermodiffusiophoretic Velocity

Thermodiffusiophoretic Velocity

For a gas mixture in the temperature gradient field, the heavier species in the mixture moves from a hot to cold region while the lighter species moves in the reverse direction. The concentration gradient, called as the Soret effect, is formed in the mixture as a result of a temperature gradient. Thus in the gas mixture, a diffusiophoretic force will be exerted on a particle in addition to the thermophoretic force. This phenomenon is called thermodiffusiophoresis.

The diffusiophoretic force is given as follows:

\[ F_{Di} = -6\pi \mu c' D_{12} |\nabla x_1|, \]  

where \( c' \), \( D_{12} \), and \( |\nabla x_1| \) are the diffusion slip factor, the coefficient of molecular diffusion, and the concentration gradient, respectively.

The diffusion slip factor \( c_{12} \) is shown as follows:

\[ c' = 0.905 \left( \frac{M_1 - M_2}{M_1 + M_2} \right) - 1.05 \left( \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2} \right), \]  

where \( M \) is the molecular weight and \( \sigma \) is the molecular diameter, respectively.

The coefficient of molecular diffusion \( D_{12} \) is shown below:

\[ D_{12} = \frac{1}{2} x_1 x_2 \left( \frac{2kT}{M} \right)^{\frac{1}{2}} d_0, \]
Appendix B: Thermodiffusiophoretic Velocity

\[ d_0 = \frac{3}{2} \left( \frac{2kT}{M} \right)^{\frac{1}{2}} \frac{E}{kT} (n_1 x_2)^{-1}, \]  
\[ (B-3) \]

\[ E = \frac{1}{8} \left( \frac{kT}{M_1 M_2 \Omega_{12}^{(1,2)}} \right), \]
\[ (B-4) \]

where \( x, k, n, \) and \( \Omega \) are the molar fraction, the Boltzmann number, the number density, and the \( \Omega \)-integral, respectively.

The concentration gradient \( \nabla \xi \) is given as follows:

\[ \nabla \xi = k_T \nabla T, \]  
\[ (B-5) \]

where \( k_T \) is the thermodiffusion ration calculated below:

\[ k_T = \left( \frac{n_1 n_2}{n_1 + n_2} \right)^2 \frac{M_1 - M_2}{M_1 + M_2} k_T^*, \]

\[ k_T^* = \frac{15}{8} \left( \frac{2\Omega_{12}^{(1,1)} - 5\Omega_{12}^{(1,2)}}{\Omega_{12}^{(2,2)}} \right) \]
\[ (B-6) \]

The thermodiffusiophoretic velocity for the gas mixture is derived by equating the balance between forces acting on a particle:

\[ F_T + F_{Di} + F_D = 0, \]

\[ V_{TD} = \frac{3 \mu C_p}{2 \rho T_{r0}} \left[ k + C_C K n + \frac{4}{3} (C_N C_K n^2 + C_N K n[k - 1]) \right] \frac{\nabla T}{\left| \nabla \xi \right|} + \left( C_C c' D_{12} \right) \left( \nabla \xi \right) \]  
\[ (B-7) \]
Effects of gas species on pressure dependence of thermophoretic velocity

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Abstract

Microgravity experiments are conducted to measure the thermophoretic velocity, and effects of gas species are investigated. Particles adopted are PMMA spheres of 2.91 μm in mean diameter, and atmospheric gases chosen are pure gases of argon, nitrogen, and carbon dioxide. The temperature gradient is set at 10 K/mm, and the pressure is set at several conditions in the range from 20 kPa to 100 kPa. Terminal velocities of particles suspended in a gas are individually measured during 0.25 s of the microgravity condition, which is achieved by a free-fall. The accuracy of the measurement is attained by accumulating data from repeated trials. Obtained experimental results are compared with theoretical predictions; a notable discrepancy is found for carbon dioxide, while the results for other two gases are consistent with predictions. Some attempts are made to fix the discrepancy: first by modifying constants and second by modifying two empirical coefficients in the theory.

1. Introduction

Thermophoresis is the phenomenon that a small particle in a gas with a temperature gradient moves toward the lower temperature side. This phenomenon is supposed to influence practically on the movement of soot particles in exhaust gas from combustors, especially near cool walls around the hot gas flow, where the temperature gradient can become as large as the order of $10^2$ K/mm. For example, the measured size-distribution of particulate matter is influenced by the temperature of the transfer tube connecting the exhaust pipe of a diesel engine to the dilution tunnel ahead of the measuring device (Yuasa et al., 2011). A soot particle is generally an aggregate of fine primary spheres. There have been some direct measurements on the thermophoretic phenomenon of such an aggregate particle. Zheng & Davis (2001) have measured the thermophoretic force acting on an aggregate of polystyrene latex spheres, and found that the force is affected by the number of primary spheres in the aggregate. Dobashi et al. (2000) and Suzuki & Dobashi (2007) have conducted direct measurements on the thermophoretic velocity of soot particles, and have revealed that the velocity is dependent not only on the macroscopic size of the soot particle but also on the aggregating condition; Suzuki & Dobashi (2007) have shown experimental results suggesting that the phenomenon is dominated by the size of primary spheres when the aggregation is coarse. Thus, the understanding of the phenomenon for a single sphere is indispensable before understanding for an aggregate.

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The thermophoretic phenomenon of a spherical particle has been studied both theoretically and experimentally. Theoretical works have been carried out by considering the boundary condition on the surface of a single spherical particle suspended in a gas with a temperature gradient. Brock (1962) derived a theoretical solution to the thermophoretic force on the particle by applying a slip boundary condition on its surface, which includes effects of the thermal slip, the viscous slip, and the temperature jump. Derjaguin & Yalamov (1965) derived the thermophoretic velocity from the Brock’s theoretical solution by equating the balance between the thermophoretic and the drag forces.

Experiments, on the other hand, have been performed by measuring the thermophoretic force or the thermophoretic velocity. Fredlund (1938) attempted a systematic experiment to examine the effect of the temperature field upon a disk suspended on a balance. The thermophoretic force and the thermophoretic velocity are measured by several methods: Millikan cell (Rosenblatt & La Mer, 1946; Saxton & Ranz, 1952; Schadt & Cadle, 1961; Jacobsen & Brock, 1965), electrodynamic balance (Li & Davis, 1995a, 1995b), precipitation in a thermoprecipitator (Schadt & Cadle, 1957; Keng & Orr, 1966), jet technique (Kousaka et al., 1976; Prodi et al., 1979; Talbot et al., 1980), and deflection of a particle suspended by a small wire (Davis & Adair, 1975; Tong, 1975).

Above experimental methods are complex in practical implementation, and as a consequence, involve numerous errors. Among those, errors caused by buoyancy are the largest problem; in a field with a temperature gradient, buoyancy induces natural convection, which influences the movement of particles and disturbs the measurement. The velocity of such natural convection is usually comparable to the thermophoretic velocity, and cannot be measured directly. To avoid this problem, some experiments have been conducted under microgravity conditions. Toda et al. (1996, 1998) performed experiments in a drop tower facility and demonstrated that the microgravity environment satisfactorily suppresses the disturbance. Prodi et al. (2006, 2007) also conducted microgravity experiments by means of a drop tower facility and/or parabolic flights. However, their reported data still seem to contain errors, possibly owing to limited trial numbers of experiments, so that those data are not sufficient to make quantitative comparison with theories (Suzuki et al., 2009a).

Recently, our group has developed a device for conducting experiments repeatedly under a microgravity environment in a very short period time, i.e. 0.3 s, by means of the free-fall method, to accumulate data of the thermophoretic velocity. It has been confirmed that satisfactory accuracy can be attained if the amount of data is sufficient for statistical treatment (Suzuki et al., 2009a). By comparing the obtained experimental results with the existing theory (Brock, 1962), two notable differences are found; one is the substantial difference for particles with high thermal conductivity, and the other the remarkable difference in the dependence of the humidity in the air (Suzuki et al., 2009b). The problem of the former difference has been solved by reconsidering the boundary condition to improve the theory (Hoshino et al., 2010a). Hoshino et al. (2010b) has derived an improved theoretical solution of the thermophoretic velocity by applying the boundary condition proposed by Lockerby et al. (2004), which includes the thermal stress slip and the higher order isothermal slip. However, even this improved theory cannot solve the problem of the latter difference. The deviation of the theory from the experiment infers that the composition of the surrounding gas mixture has some unknown influence on the phenomenon.

Before analyzing the influence of the composition of the gas mixture, it is indispensable to know the characteristics of the phenomenon for each gas component. Thus, in this study, the characteristics of the thermophoresis for several gas species, i.e., argon, nitrogen, and carbon dioxide are experimentally examined. The first one is chosen as the reference, and the latter two are chosen as the major components of exhaust gas from combustors. Water vapor is the clue of starting this work, but it is excluded owing to technical reasons on conducting the experiment.

2. Theory for thermophoretic velocity

The theory adopted in this work is based on the works by Hoshino et al. (2010b) and by Chang & Keh (2012). The thermophoretic velocity is calculated from the balance between the thermophoretic and the drag forces, the equation of which is shown as below

\[ V_T = \frac{2\mu C_F [C_F(k+C_F Kn) + C_D(C_M C_F Kn^2 + C_M Kn(k-1))]}{\rho T_{j0} [1+3C_M Kn + h(1+2k+2C_F Kn)]} |\nabla T|, \]

where \( \mu, |\nabla T|, \rho, T_{j0}, C_F, k, Kn, h, C_M, C_D, C_S, \) and \( C_D \) are the viscosity, the temperature gradient, the density of the surrounding gas, the reference temperature, the slip correction factor of the drag force for rarefied condition, the gas-to-particle thermal conductivity ratio, Knudsen number, the term of higher order isothermal slip, and constants for slip flow, temperature jump, thermal creep, and thermal stress slip, respectively. Here, the reference temperature \( T_{j0} \) is defined as the supposed gas temperature at the center of the particle in the given temperature field without the existence of the particle, and Knudsen number \( Kn \) is the ratio of the mean-free-path \( l \) to the particle radius. The mean-free-path is calculated from the following equation (Kennard, 1938):

\[ l = \frac{\mu}{0.499P} \sqrt{\frac{\pi RT_{j0}}{8}}, \]

where \( P \) and \( R \) are the pressure and the gas constant, respectively. The slip correction factor \( C_F \) adopted in this work is Cunningham’s correction factor \( C_F = 1 + A Kn \), where \( A = 1.257 + 0.4exp(-1.10/Kn) \) (Talbot et al., 1980). The term \( h \) of
Higher order isothermal slip is written as below

\[ h = \frac{9}{2\pi} \Pr Kn^2 (1 - \gamma^{-1}) \]  

(3)

where \( Pr \) and \( \gamma \) are Prandtl number and the specific heat ratio, respectively.

Eq. (1) is basically identical to both of those proposed by Hoshino et al. (2010b) and Chang & Keh (2012). If constant values \( C_s = 0.75 \) and \( C_H = 1.00 \) (Lockerby et al., 2004) are applied, the equation becomes just the same as the one given by Hoshino et al. (2010b). On the other hand, the equation becomes the same as the one given by Chang & Keh (2012) by neglecting the term \( h \), which is usually negligible in the range of \( Kn \leq 0.1 \), and applying Basset’s correction (Basset, 1961). \( C_{fb} = \frac{(1 + 3C_M Kn)}{(1 + 2C_M Kn)} \), instead of the Cunningham’s correction for the slip correction factor \( C_f \). The Basset’s correction is applicable in \( Kn \leq 0.1 \) while the Cunningham’s one in \( Kn \leq 1 \), and both are basically identical in those applicable ranges.

Two constants \( C_M \) and \( C_T \) are calculated as follows from two empirical coefficients, i.e., the tangential momentum accommodation coefficient \( \alpha_m \) and the thermal accommodation coefficient \( \alpha_t \)

\[ C_M = \frac{2 - \alpha_m}{\alpha_m} \]  

(4)

\[ C_T = \frac{15}{8} \left( \frac{2 - \alpha_t}{\alpha_t} \right) \]  

(5)

These accommodation coefficients represent the magnitude of the momentum and energy exchange in the collision between gas molecules and the particle (Kennard, 1938; Shen, 2010), which are defined as below

\[ \alpha_m = \frac{M_i - M_f}{M_i - M_s} \]  

(6)

\[ \alpha_t = \frac{E_i - E_r}{E_i - E_s} \]  

(7)

where \( M \) and \( E \) are average tangential components of the momentum and the energy flux normal to the particle surface, respectively, of gas molecules, and subscripts \( i, r, \) and \( s \) refer to incident molecules, molecules reflecting from the surface, and molecules leaving the surface in equilibrium with the surface, respectively. For example, \( \alpha = 1 \) corresponds to the situation that incident molecules achieve complete equilibrium with the particle surface before leaving, while \( \alpha = 0 \) corresponds to the situation of complete specular reflection.

In many cases, values of accommodation coefficients are simply assumed to be unity (Suzuki et al., 2009b; Hoshino et al., 2010a). There are some experimental measurements, e.g., Thomas & Lord (1974) and Douglas (1982), who measured accommodation coefficients on steel spheres and gas covered tungsten tube, respectively. Although those experiments report values different from unity under some conditions, yet there are no commonly accepted values.

3. Experimental

3.1. Apparatus

Fig. 1 shows a schematic of the apparatus, which includes a drop tower, a measuring unit, and a damping cushion. The measuring unit is hung at the top of the drop tower by an electric magnet. The unit starts falling when the electric

![Schematic of apparatus.](image-url)
magnet is deactivated. The falling distance is 0.6 m, which corresponds to the duration time of the microgravity condition at 0.3 s.

Fig. 2 shows a variation of the gravity level during an experiment. The gravity level is measured by the G-sensor attached on the measuring unit. The duration time of the free-fall is about 0.3 s as seen in the figure. When the electric magnet is deactivated at $t=0$, the gravity level in $z$-axis changes from $-1.0 \text{ G}$ to $+0.4 \text{ G}$ once, possibly owing to the vibrational motion of the frame of the unit, and then decays as time passes until the unit reaches the cushion. The range between $±0.1 \text{ G}$ in gravity level is regarded as the microgravity condition in this work, the duration time of which is about 0.25 s.

Fig. 3 shows a cross-section of the pressure vessel, the main part of the measuring unit. Placed in the vessel is a cylindrical thin chamber of 1.5 mm in height between two opposing aluminum plates. The upper plate, initially at the room temperature, is kept cool by moving it 15 mm away from the bottom plate while the latter is under the process of heating to raise its temperature. Keeping this arrangement, the vessel is evacuated by a pump. When the bottom plate reaches the appointed temperature, the upper plate is put in place where the distance between the plates is 1.5 mm. Then, the vessel is filled with the test gas. When the temperature of the upper plate becomes the target temperature, both electromagnetic valves at the inlet and the outlet are simultaneously opened for 0.01 s such that some particles are blown into the chamber from a particle reservoir upstream the inlet. The magnet holding the measuring unit is deactivated 0.84 s after the closure of both valves. The disturbances caused by the blow are expected to cease within this period before the drop; since it is larger than both characteristic times for the flow field and the temperature field to reach the steady state; those characteristic times are expressed as $t_f = (\delta^2 / 2 \nu)$ and $t_t = (\delta^2 / 2 \alpha)$, respectively, where $\delta$, $\nu$, and $\alpha$ are the distance between two plates, the kinematic viscosity, and the thermal diffusivity, respectively (Toda et al., 1996, 1998). The largest values of those for present conditions are estimated at 0.06 s and 0.05 s. Those blown particles are illuminated by the laser.
beam introduced into the chamber to produce scattering lights, the image of which is recorded by a high-speed camera under the condition of the frame rate at 200 fps and of the shutter speed at 2 ms.

3.2. Sample particles and experimental condition

Sample particles used in this work are PMMA sphere particles from Sekisui Plastics Co., Ltd. These are chosen since the size is quite uniform; Fig. 4 shows the probability density distribution of the particle diameter $d$ measured from SEM images. The mean diameter and the standard deviation are 2.91 $\mu$m and 0.09 $\mu$m, respectively. The thermal conductivity is measured by the manufacturer at 0.21 W/(mK).

Experiments are conducted under the microgravity condition with the test gas of argon, nitrogen or carbon dioxide. The target temperature and the temperature gradient are 313 K and 10 K/mm, respectively. The temperature field during the experiment is monitored by measuring temperatures at two points in the vessel: one is in the top plate and the other in the bottom one. The temperature field is controlled based on these measured temperatures. Before making experiments, the preparatory measurement for the temperature field is conducted by inserting two more thermocouples suspended in the chamber at different heights. Measuring two points is sufficient because the linearity of the temperature field has already been confirmed by the Mach–Zehnder interferometry (Toda et al., 1996, 1998). The relation between these temperatures at monitoring points and the actual temperature field in the chamber is examined from this preparatory measurement. When conducting thermophoresis experiments, additional two thermocouples are removed; the temperature field is controlled based on the temperatures at those two monitoring points.

Various pressure conditions are chosen from 20 kPa to 100 kPa. The thermophoretic velocity for each particle is individually measured, and the mean value and the 95% confidence interval for each experimental condition are statistically obtained.

4. Results

Fig. 5 shows examples of the movement of particles during a free-fall in the surrounding gas of argon at 20 kPa. The measurement of the velocity should be taken at a fixed temperature since the thermophoretic velocity is dependent on not only the temperature gradient but also the temperature itself. The velocity of each particle is measured by tracing its movement while it travels within the range of the temperature between 313 ± 2 K. It is seen from the figure that the velocity of each particle can be considered as constant in the range. The velocity is constant also in other two gases.

Table 1 shows the statistical data for the tested pressure conditions with the argon gas. It is noted that the 95% confidence interval indicates not the range of data scattering but the range of expected mean value of the population. It is seen that the confidence interval is roughly estimated at around 0.01 mm/s to 0.02 mm/s. The ratio of the confidence interval to the mean value is only 3% for the pressure at 20 kPa. As the pressure increases, the velocity decreases, and as a consequence, the ratio tends to increase.

Fig. 6 shows the thermophoretic velocity for each gas. The white rectangle (□), the black circle (●), and the white circle (○) represent experimental values for argon, nitrogen, and carbon dioxide, respectively. Error bars in the figure indicate the 95% confidence interval. The solid line is the prediction calculated by assuming constants to be identical to those in the previous work (Hoshino et al., 2010a), namely, $C_M$, $C_T$, $C_s$, and $C_N$ to be 1.000, 1.875, 0.750, and 1.000, respectively.

For argon and nitrogen, predictions are in good agreement with experiments throughout all the tested pressure conditions; the solid line runs through the range of the confidence interval at every tested pressure. On the other hand, for carbon dioxide, discrepancy is seen at low pressure conditions; the prediction is within the range of the confidence interval.
only when the pressure is at 100 kPa or 80 kPa. As the pressure decreases, discrepancy becomes notable; the experimental value at 20 kPa is 0.182 mm/s, which is 70% of the theory.

5. Discussion

It is quite interesting that the discrepancy is seen only for carbon dioxide. Also, previous work (Hoshino et al., 2010a) has shown satisfactory accordance between the prediction and the experiment for the air. As noted earlier, the theory contains four constants, \( C_M \), \( C_T \), \( C_S \), and \( C_H \), and the former two of these are calculated from accommodation coefficients \( a_m \) and \( a_t \). These constants and coefficients have been determined empirically; different researchers have given different proposed values. Attempts should be made for finding suitable values to fit the prediction to these experimental results.

The first attempt is done by applying proposed values from references, which are shown in Table 2. The set of values noted as Case 1 is from the paper by Hoshino et al. (2010a), and those as Case 2 and 3 are both from the paper by Chang & Keh (2012). Fig. 7 shows predictions together with experimental results. The vertical axis represents the reduced thermophoretic velocity, which is a dimensionless parameter defined as follows:

\[
V_{TR} = \frac{V_T T_0}{\nu \nabla T},
\]

where \( \nu \) is the kinematic viscosity of the gas. This parameter is often used when making comparison between different conditions. Lines labeled as Prediction 1, 2, and 3 are those predicted from Eq. (1) by applying values of Case 1, 2, and 3, respectively. For both argon and nitrogen, the best fit to experimental results is Prediction 1, and then Prediction 3 follows; the velocity predicted from Case 3 is higher than the experiment at \( Kn = 0.25 \) for both two gases, whereas that from Case 1 is within the error bar throughout all the measured range of \( Kn \). For carbon dioxide, on the other hand, there is no line predicting satisfactorily the experimental result at \( Kn = 0.15 \), though Prediction 3 seems to be better than Prediction 1. Prediction 2 can be omitted from the discussion since it obviously disagrees with the experiment for all gases.

Provisionally, Case 1 is chosen as the best among three cases; Case 3 is not good enough for high \( Kn \) for all tested gases in comparison to Case 1. Choosing Case 1 instead of Case 3 is also consistent with the previous work, in which experiments have been conducted in the air under the atmospheric pressure for particles of PMMA, alumina, and Zn (Suzuki et al., 2009b); as shown in Table 3, significant differences are seen between Case 3 and experimental results for alumina and Zn, whereas agreements are satisfactory for all particles with Case 1. In the theory, only the radius and the thermal conductivity are considered as particle-related parameters. It would be reasonable to assume that coefficients for CO\(_2\) are different from those for other gases; Sharipov (2004) has reviewed related papers and summarized accommodation coefficients for various gases, in which CO\(_2\) values are notably different from Ar and N\(_2\).
Fig. 6. Pressure dependence of thermophoretic velocity in each gas. Error bars represent the confidence interval (95%). Predictions are calculated under the assumption of $C_M=1.000$, $C_T=1.875$, $C_S=0.750$, and $C_H=1.000$: (a) argon, (b) nitrogen and (c) carbon dioxide.

Table 2

<table>
<thead>
<tr>
<th>Case</th>
<th>$C_M$</th>
<th>$C_T$</th>
<th>$C_S$</th>
<th>$C_H$</th>
<th>$x_m$</th>
<th>$x_H$</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>1.000</td>
<td>1.875</td>
<td>0.750</td>
<td>1.000</td>
<td>(1.000)$^a$</td>
<td>(1.000)$^a$</td>
<td>Hoshino et al. (2010a)</td>
</tr>
<tr>
<td>Case 2</td>
<td>1.140</td>
<td>2.180</td>
<td>1.170</td>
<td>1.000</td>
<td>(0.935)$^a$</td>
<td>(0.925)$^a$</td>
<td>Chang &amp; Keh (2012)</td>
</tr>
<tr>
<td>Case 3</td>
<td>1.140</td>
<td>2.180</td>
<td>1.170</td>
<td>3.000</td>
<td>(0.935)$^a$</td>
<td>(0.925)$^a$</td>
<td>Chang &amp; Keh (2012)</td>
</tr>
</tbody>
</table>

$^a$Values in bracket are from Eqs. (4) and (5) (not directly given in the reference).
The second attempt is done by changing the accommodation coefficients from Case 1 by means of the least square fit. The residue $R$ representing the discrepancy between the experiment and the prediction is defined as follows:

$$R \equiv \sum_{j=1}^{n} (V_{ej} - V_{pj})^2, \quad (j = 1, \ldots, n)$$

Fig. 7. Comparison between predictions and experiment in each gas. Error bars represent the confidence interval (95%). Predictions are calculated using constants in Table 2: (a) argon, (b) nitrogen and (c) carbon dioxide.

The second attempt is done by changing the accommodation coefficients from Case 1 by means of the least square fit. The residue $R$ representing the discrepancy between the experiment and the prediction is defined as follows:

$$R \equiv \sum_{j=1}^{n} (V_{ej} - V_{pj})^2, \quad (j = 1, \ldots, n)$$
where $V_e$, $V_p$, and $n$ are thermophoretic velocities of the experiment and the prediction, and the number of the tested pressure conditions, respectively, and the subscript $j$ refers to each tested condition. The values for $C_S$ and $C_H$ are unchanged from the Case 1, and those for $C_M$ and $C_T$ are calculated from Eqs. (4) and (5), respectively, with accommodation coefficients $a_m$ and $a_t$ determined such that the residue $R$ becomes the lowest.

Table 4 shows the result of the least square fit for carbon dioxide. The best fit is attained with the coefficients noted as “without limitation” in the table. It should be noted that both values are clearly above unity, which is curious when interpreting the physical meaning of those values. Based on the kinetic theory, both those coefficients should lie between 0 and 1 (Brock, 1962; Kennard, 1938; Shen, 2010). The Cercignani-Lampis (CL) model provides a more physical description of the gas-surface interaction (Sharipov, 2003, 2004), which allows $a_m$ to vary between 0 and 2 while $a_t$ remains between 0 and 1. In this CL model, $a_m$ can exceed unity when the surface is rough. Considering this, additional work is done for the fitting by applying the limitation of $0 \leq a_t \leq 1$ and $0 \leq a_m \leq 2$, the result of which is shown as “with limitation” in the table. In this case, $R$ becomes the lowest when both the coefficients are bellow unity even $a_m$ is allowed to exceed it.

Fig. 8 shows the comparison of predictions based on these values. It is seen that both predictions are in good agreement with the experiment within the range of the tested conditions. The difference between these two predictions is notable only under the condition of higher Knudsen number than the current work.

In order to interpret the different coefficients for the carbon dioxide, authors have found two different points of view from literatures so far; one is the molecular weight, and the other the diameter of a molecule. The former is based on the work by Gronych et al. (2004), who have conducted experiments to find out that the tangential momentum accommodation coefficient increases as the molecular weight of the gas decreases. This may be related to the influence of gas species. However, applying
this effect for the explanation of the present results would not be appropriate; the molecular weight of argon is closer to carbon dioxide than nitrogen, while the thermophoretic characteristic of the argon is much closer to the nitrogen rather than the carbon dioxide.

The latter is based on the work by Arya et al., (2003), who have conducted a molecular simulation for a wall-slip phenomenon in rarefied gases. According to their result, the tangential momentum accommodation coefficient decreases as the collision diameter of the molecule increases. Among the gases used in this work, carbon dioxide has a notably larger collision diameter; based on the rigid sphere model, the diameter of argon, nitrogen, and carbon dioxide are 3.659 Å, 3.784 Å, and 4.643 Å, respectively (Ivchenko et al., 2007). This can explain qualitatively the results in this work if the coefficients “with limitation” are adopted for the carbon dioxide and those at unity are adopted for other two gases. The thermal accommodation coefficient “with limitation” is also consistent with the work by Winkler et al. (2004), in which the coefficient $\gamma_2$ is reported to be approximately unity for water vapor, a polyatomic gas having a large collision diameter. The constants and coefficients, however, cannot be determined from the results in this work, since several combinations of those are possible to fit predictions to experimental results.

6. Conclusions

In this study, effects of gas species on the thermophoretic velocity are investigated, and following results are obtained:

1. Experimentally-obtained pressure dependence of the thermophoretic velocity for both argon and nitrogen are quantitatively in good agreement with the theory, while notable discrepancy is seen for carbon dioxide, if values for the constants $C_M$, $C_T$, $C_S$, and $C_D$ to be 1.000, 1.875, 0.750, and 1.00 are applied to the theory.
2. Values of those empirical constants proposed in some references were applied to the theory and compared with experiments; none of those has reduced the notable deviation for the carbon dioxide at $Kn=0.15$.
3. The coefficients are calculated by means of the least square fit. The obtained values can be interpreted qualitatively by considering the effect of the molecular diameter on the tangential momentum accommodation coefficient. The proposed values of $C_M$, $C_T$, $C_S$, and $C_D$ for carbon dioxide are 1.378, 1.875, 0.750, and 1.00, although further discussion will be required for the determination of the values.

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References


Measurement of thermophoretic parameters for binary gas mixtures

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A B S T R A C T

The thermophoretic velocity of a particle in a gas mixture is measured by means of a microgravity experiment. Adopted particles are PMMA spheres of 2.91 μm in mean diameter, and gas mixtures chosen are argon–nitrogen, argon–carbon dioxide, and nitrogen–carbon dioxide. The temperature gradient and the pressure are 60 K/mm and 70 kPa, respectively. Terminal velocities of particles suspended in a gas are individually measured. The tangential momentum accommodation coefficient is estimated from experimental result by assuming the thermal accommodation coefficient at unity, and it is compared with predictions calculated from values of composing pure gases by means of some methods given in references; among those methods, the intermolecular-interaction-based method is found to be the best for all mixture conditions.

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1. Introduction

A small particle suspended in a quiescent gas with a temperature gradient will move towards the lower temperature side. This phenomenon, called thermophoresis, influences the soot formation process in combustion fields (Dobashi et al., 2000). The phenomenon is also applied in a thermopositor used for collecting submicron size particles in a manufacturing process of optical fibers (Morse et al., 1985). Quantitative understanding of the phenomenon is indispensable for controlling the movement of particles in those systems.

Recent experiments by means of the free-fall method, which suppresses the buoyancy-induced disturbance to attain satisfactory accuracy (Toda et al., 1996, 1998; Suzuki et al., 2009a), have exhibited a remarkable difference from the theory when the air contains moisture (Suzuki et al., 2009b). This deviation infers the existence of unknown influence of gas species and/or gas mixing on the phenomenon that has not been taken into consideration in the theory (Hoshino et al., 2010b; Chang & Keh, 2012), which gives reasonable prediction in the case of dry air.

Influence of gas species on the thermophoretic phenomenon has been investigated in the previous work (Mohd Azahari et al., 2012); the thermophoretic velocity has been measured for pure gases of argon, nitrogen, and carbon dioxide. Satisfactory agreement with the theory is seen in cases of argon and nitrogen, while a noticeable deviation from the theory
is seen in the case of carbon dioxide. It has been demonstrated in the paper that this deviation can be corrected by modifying two empirical constants, i.e., the tangential momentum accommodation coefficient and the thermal accommodation coefficient, both of which are involved in the theory and usually assumed to be unity (Li & Davis, 1995a; Prodi et al., 2007; Suzuki et al., 2009b; Hoshino et al., 2010a).

A problem arises here for the treatment of accommodation coefficients of a mixture of pure gases having different values, e.g., the mixture of nitrogen and carbon dioxide. It should better be confirmed even when the mixture is composed of pure gases having the same coefficient value, e.g. the mixture of argon and nitrogen, since there is no widely-accepted method so far for evaluating the coefficients of the mixture from the values of composing pure gases.

There are some theoretical and experimental researches dealing with one of those coefficients of gas mixtures, although none of those gives explicit discussion for both the coefficients together. Mikami et al. (1966) have measured heat-transfer coefficients of a sphere in hydrogen–nitrogen and helium–nitrogen mixtures by utilizing a thermistor, and have compared those results with an analytical solution based on the Maxwell’s model; good agreement has been shown by applying a linearly-dependent relation of the accommodation coefficient to the gas concentration. Wise et al. (1966) have conducted experiments to measure the heat transfer with a conductivity cell in a gas mixture of atomic and molecular oxygens. The coefficient for pure atomic oxygen is estimated from extrapolation of experimentally-determined thermal accommodation coefficients of mixtures containing small amount of atomic oxygen. Ivchenko et al. (1997, 2007) have studied the slip phenomenon in the flow of a gas mixture to develop an analytical solution for planar transport problems. Bentz et al. (1999) have performed experiments with a spinning rotor gauge to measure the slip flow constant, which is related to the tangential momentum accommodation coefficient, for helium–argon, helium–nitrogen, and helium–neon mixtures. Bentz et al.’s experimental results exhibit a nonlinear dependence on the mixture concentration; it seems to agree qualitatively well with the prediction from the Ivchenko et al.’s solution, though they have written of it “there are large differences”. Further investigation will be needed in order to determine the appropriate method for evaluating mixture’s coefficients.

In this study, thermophoretic parameters are experimentally investigated for several gas mixtures, i.e., argon–nitrogen, argon–carbon dioxide, and nitrogen–carbon dioxide. The dependence of the tangential momentum accommodation coefficient on the concentration of the mixture is examined from the measured thermophoretic velocity.

2. Factors affected by gas mixing

The problem for a gas mixture is the treatment of two empirical parameters, i.e., tangential momentum- and thermal-accommodation coefficients, which characterize the interaction between the solid surface of the particle and surrounding gas molecules. In the gas mixture, a diffusiophoretic force will be exerted on a particle in addition to the thermophoretic force since the temperature gradient causes a concentration gradient. The thermodiffusiophoretic velocity for a gas mixture is shown as follows

\[
V_{TD} = \frac{3\mu C_c}{2\rho T_{fo}} \left[ \left( k + C_f \frac{k_f}{\rho} \right) + \frac{(4/3)\rho C_M C_f k_f}{(2\pi)^{1/2}} + \frac{C_M k_f}{k_f} \right] \sum n \left[ \nabla T \right] + C_c D_{fo} \left| \nabla x \right|, 
\]

where \( \mu \), \( \rho \), \( T_{fo} \), \( C_c \), \( k \), \( Kn \), \( Pr \), \( \gamma \), \( C_M \), \( C_f \), \( D_{fo} \), and \( \nabla x \) are the viscosity, the temperature gradient, the density of the gas, the reference temperature, the Cunningham's correction factor, the gas-to-particle thermal conductivity ratio, Knudsen number, Prandtl number, the specific heat ratio, the constant for slip flow, the constant for temperature jump, the diffusion slip factor, the coefficient of molecular diffusion, and the concentration gradient, respectively. Here, the reference temperature \( T_{fo} \) is defined as the supposed gas temperature at the center of the particle in the given temperature field without the existence of the particle. The first term in the right hand side of Eq. (1) is the thermophoretic term, which is the same as in the previous work (Mohd Azahari et al., 2012). The second term is the diffusiophoretic term (Waldmann, 1959).

The density \( \rho \), the viscosity \( \mu \), and the thermal conductivity of gas \( k_f \) for a binary gas mixture are calculated as follows (Richard, 1964)

\[
\rho = \frac{P_1}{R_1 T} + \frac{P_2}{R_2 T},
\]

\[
\mu = \frac{\sum n}{\sum n} \frac{\mu_j}{\sum n} \phi_{ij}(x_n/x_j),
\]

\[
k_f = \frac{\sum n}{\sum n} \frac{k_{ij}}{\sum n} \phi_{ij}(x_n/x_j),
\]

\[
\gamma = \frac{C_M}{C_f} + \frac{C_M}{C_f} - 1.
\]
here,

\[ \phi_{jm} = \frac{(1 + (\mu_j/\mu_m)^{1/2})(M_n/M_j)^{1/4})^2}{2\sqrt{2(1 + (M_j/M_n))^{1/2}}} \]  

(5)

\[ \phi_{jm} = \frac{1}{4} \left( 1 + \left[ \frac{\mu_j}{\mu_n} \left( \frac{M_n}{M_j} \right)^{3/4} \frac{1 + (C_j/T)}{1 + (C_n/T)} \right]^{1/2} \right)^2 \left( \frac{1 + (C_{jm}/T)}{1 + (C_j/T)} \right). \]  

(6)

\[ C_{jm} = (C_jC_n)^{0.5}, \]  

(7)

where \( R \) and \( C \) are the gas constant and Sutherland constant, respectively. Subscripts \( j \) and \( n \) refer to the number of each gas species and the total number, respectively. Parameters \( c_j, D_{12}, \) and \( \nabla x_{ij} \) are calculated by complex equations given by Kihara (1975) and Ivchenko et al. (2007).

Constants \( C_M \) and \( C_T \) in Eq. (1) are the ones containing the tangential momentum accommodation coefficient \( \alpha_m \) and the thermal accommodation coefficient \( \alpha_t \), respectively, which are written as follows:

\[ C_M = \frac{2-\alpha_m}{\alpha_m}, \]  

(8)

\[ C_T = \frac{15}{8} \left( \frac{2-\alpha_t}{\alpha_t} \right). \]  

(9)

In order to apply this same formula to a gas mixture, those coefficients of the mixture should be determined from values of each pure gas components.

In this work, only the tangential momentum accommodation coefficient is derived from experimental results, and the other one, the thermal accommodation coefficient, is assumed to be unity on the basis of results in the previous work (Mohd Azahari et al., 2012), in which all the thermal accommodation coefficients have been estimated at unity for the same pure gases adopted in this work.

The authors have found three different methods for the calculation of the tangential momentum accommodation coefficient of a mixture, which are explained below

i. Energy-balance-based method
The first one is given by Mikami et al. (1966), who derived the tangential momentum accommodation coefficient for a gas mixture from the energy balance as below

\[ \alpha_{m,mix} = \sum_{j=1}^{n} y_j \alpha_{mj}/\sqrt{M_j} \]  

(10)

where \( x \) and \( M \) are the molar fraction and the molecular weight, respectively, and subscript \( j \) refers to the number of each gas species.

ii. Mass-fraction-based method
The second one is found in the ANSYS FLUENT theory guide (2009), in which both the accommodation coefficients of a mixture are calculated simply by taking the mass-fraction weighted average of each gas species as follows:

\[ \alpha_{mix} = \sum_{j=1}^{n} y_j \alpha_j, \]  

(11)

where \( y_j \) and \( \alpha \) are the mass fraction of each gas species \( j \) and each accommodation coefficient.

iii. Intermolecular-interaction-based method
The last one can be derived from equations given by Ivchenko et al. (2007). The constant \( C_M \) for the slip-flow of a gas mixture was derived using the first-order Chapman–Enskog approximation and the Maxwell method as below

\[ C_M = \frac{5}{16} \frac{P\sqrt{x_1x_1M_1 + x_2x_2M_2} (2-\alpha_{1m})x_1b_1 + (2-\alpha_{2m})x_2b_2}{\mu \alpha_{1m}^{1/2}x_1^{1/2} + \alpha_{2m}^{1/2}x_2^{1/2} \sqrt{M_1 + M_2}}. \]  

(12)

where \( P \) and \( b \) are the pressure and the transport coefficient (Ivchenko et al., 2007) of viscosity corresponding to arbitrary models of the intermolecular interaction, respectively, and subscripts 1 and 2 refer to components of the binary gas mixture. Here, it is noted that the literature adopts the following equation instead of Eq. (8):

\[ C_M = \frac{5}{16} \frac{2-\alpha_m}{\alpha_m}. \]  

(13)

This equation is basically identical to the Eq. (8): the additional coefficient of \( 5\pi/16 \) to the Eq. (8) makes the resultant value only 2\% less. By equating Eqs. (12) and (13), the tangential momentum accommodation coefficient of the mixture
can be formulated as a function of the coefficients of two pure gases as follows:

\[ \alpha_{m,\text{mix}} = 2 \left(1 + \frac{P\sqrt{x_1M_1 + x_2M_2}(2-\alpha_{1m}x_1b_1 + (2-\alpha_{2m}x_2b_2)}{\alpha_{1m}x_1\sqrt{M_1} + \alpha_{2m}x_2\sqrt{M_2}} \right)^{-1}. \]  

(14)

3. Experimental

The experimental configuration used in this work is the same as the previous (Mohd Azahari et al., 2012) except that a gas mixer is introduced. A mixture of two gases is created before it is supplied into the pressure vessel, in which particles are supplied to measure the thermophoretic velocity. The mixer is evacuated by a pump before two gases feed into it. Gas mixtures in various ratios are prepared by adjusting the partial pressure of each gas. The gases used in this work are argon, nitrogen, and carbon dioxide.

Sample particles used in this work are the same spherical PMMA particles used in the previous. As it is shown in Table 1, those particles are quite uniform in size. The mean diameter and the thermal conductivity of these particles are 2.91 μm and 0.21 W/(m K), respectively. The pressure, the reference temperature, and the temperature gradient of the surrounding gas are 70 kPa, 338 K, and 60 K/mm, respectively. The measurement is performed by means of the free-fall method. Drop experiments are conducted repeatedly to accumulate enough amounts of data. Details are described in the previous paper (Mohd Azahari et al., 2012). It is noted that the temperature gradient in the present work is increased, and the reference temperature is changed accordingly, from the previous condition such that the thermophoretic velocity increases and the measurement accuracy increases.

The tangential momentum accommodation coefficient \( \alpha_m \) for each experimental datum is calculated from Eqs. (1), (8), and (9) by assuming \( \alpha_i = 1 \), and the mean value and its 95% confidence interval are statistically obtained for each experimental condition. It is noted that those data for pure gases include experimental results from the previous work (Mohd Azahari et al., 2012). The experimentally-obtained coefficient \( \alpha_m \) of a gas mixture is compared with the ones calculated from values of pure gases by Eqs. (10), (11), and (14).

4. Results

Fig. 1 shows measured velocities for argon–nitrogen, argon–carbon dioxide, and nitrogen–carbon dioxide mixtures. Lines represent predictions by above-mentioned methods, which will be explained later. Error bars in the figure indicate the 95% confidence interval for the mean.

Nitrogen exhibits the highest thermophoretic velocity, followed by argon and carbon dioxide; the thermophoretic velocities for pure nitrogen, argon, and carbon dioxide are 0.880 mm/s, 0.673 mm/s, and 0.273 mm/s, respectively. For the mixture of argon–nitrogen, the thermophoretic velocity increases as the concentration of nitrogen increases. For mixtures of argon–carbon dioxide and nitrogen–carbon dioxide, the thermophoretic velocity decreases as the concentration of carbon dioxide increases.

Fig. 2 shows tangential momentum accommodation coefficients of pure gases of argon, nitrogen, and carbon dioxide. White and black symbols represent values from the previous (Mohd Azahari et al., 2012) and this works, respectively. Solid lines indicate mean values among all pressure conditions. Coefficients from this work seem to be consistent with those from the previous.

Table 2 shows mean values of the coefficient from all the pressure conditions. Coefficients for argon and nitrogen are almost the same: 0.987 and 0.986, respectively. That for carbon dioxide exhibits a notably low value at 0.821.

Table 3 shows the tangential momentum accommodation coefficient calculated from experimental results. Error ranges indicate the 95% confidence interval for the mean. For argon–carbon dioxide and nitrogen–carbon dioxide, it is seen that the coefficient is dependent on the gas concentration. For the mixture of argon–nitrogen, the slightly increase up to 1% is noticeable in the mean value. However, it is difficult to make a statistically significant since the mean values are within the error range.

Fig. 3 shows comparisons of the tangential momentum accommodation coefficient of experiments and predictions from Eqs. (10), (11), and (14). In cases of gas mixtures of argon–nitrogen and argon–carbon dioxide, predictions by the energy-balance and the

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of PMMA particles.</td>
</tr>
<tr>
<td>Mean (μm)</td>
</tr>
<tr>
<td>2.91</td>
</tr>
</tbody>
</table>
mass-fraction methods are almost identical. On the other hand, in the case of nitrogen–carbon dioxide, predictions by the energy-balance and the intermolecular-interaction methods give close results while the mass-fraction method is distinguishable from others. Although there is slightly difference between methods, all predictions agree satisfactorily with the experimental result for argon–nitrogen. For argon–carbon dioxide, disagreements are seen between the experimental result and predictions by the energy-balance and the mass-fraction methods. For nitrogen–carbon dioxide, slightly disagreement is noticeable between the mass-fraction method and the experimental result at 25% carbon dioxide. The intermolecular-interaction method seems to be the best for all those mixture conditions.

Fig. 1. Thermophoretic velocity for each gas mixture. Error bars represent the confidence interval (95%) for the mean. (a) Ar–N₂, (b) Ar–CO₂ and (c) N₂–CO₂.

Comparisons of thermophoretic velocities between predictions and experiments for gas mixtures are shown in Fig. 1. Although the difference is not so remarkable, the intermolecular-interaction method seems to be the best among compared three methods; the energy-balance and the mass-fraction methods underestimate the velocity for argon-carbon dioxide.
Fig. 2. Tangential momentum accommodation coefficient for each pure gas. Error bars represent the confidence interval (95%) for the mean. (a) Argon, (b) Nitrogen and (c) Carbon dioxide.

Table 2

<table>
<thead>
<tr>
<th>Gas</th>
<th>$\alpha_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argon</td>
<td>0.987</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>0.986</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>0.821</td>
</tr>
</tbody>
</table>
Table 3
Estimated tangential momentum accommodation coefficient $\alpha_m$ for each gas mixture.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Ar:N$_2$</th>
<th>Ar:CO$_2$</th>
<th>N$_2$:CO$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00:0.00</td>
<td>0.987 ± 0.028</td>
<td>0.987 ± 0.028</td>
<td>0.986 ± 0.026</td>
</tr>
<tr>
<td>0.75:0.25</td>
<td>0.989 ± 0.027</td>
<td>0.991 ± 0.025</td>
<td>0.965 ± 0.028</td>
</tr>
<tr>
<td>0.50:0.50</td>
<td>0.993 ± 0.031</td>
<td>0.949 ± 0.029</td>
<td>0.905 ± 0.022</td>
</tr>
<tr>
<td>0.25:0.75</td>
<td>0.992 ± 0.026</td>
<td>0.877 ± 0.022</td>
<td>0.846 ± 0.033</td>
</tr>
<tr>
<td>0.00:1.00</td>
<td>0.986 ± 0.026</td>
<td>0.821 ± 0.031</td>
<td>0.821 ± 0.031</td>
</tr>
</tbody>
</table>

**Fig. 3.** Comparison of tangential momentum accommodation coefficient for each gas mixture. Error bars represent the confidence interval (95%) for the mean. (a) Ar-N$_2$, (b) Ar-CO$_2$ and (c) N$_2$-CO$_2$. 

5. Discussion

A problem in the determination of coefficients of a gas mixture is the uncertainty of those values of composing pure gases. Different researchers have proposed different values for a same pure gas. Table 4 shows comparisons of coefficients between several references for the concerned pure gases; the values obtained in this work are also shown. These differences in the table infer the possibility that the accommodation coefficient is dependent on the adopted method and/or the material of the solid. The surface condition of the solid wall may also have some influence: Thomas and Lord (1974) have measured accommodation coefficients for polished and rough surfaces of steel spheres to find some difference as shown in the table.

This problem should be considered when making quantitative comparison between values from different research works. With regard to the current and its preceding works, all experimental data are obtained from the same method and the same kind of particles. Thus, the comparison given in this work will not be concerned directly by the problem.

Another problem is the assumption that the thermal accommodation coefficient is always at unity regardless of the mixing condition. This does not concern when applying energy-balance and mass-fraction based methods, since both these methods do not alter the coefficient of the mixture when both the pure gases have the same value at unity. The intermolecular-interaction-based method, on the other hand, may change the value of the thermal accommodation coefficient depending on the concentration as is the case of the tangential momentum accommodation coefficient. Authors did not find so far any reference that shows the way to calculate the thermal accommodation coefficient on the intermolecular interaction basis. Although there are such vulnerable points in the treatment of coefficients, the current work would be informative for a practical use in predicting the thermophoretic velocity for gas mixtures. Experimental results are satisfactorily predicted when the thermal accommodation coefficient is assumed to be unity and the other coefficient is calculated from pure gas values by means of the intermolecular-interaction-based method.

6. Conclusions

In this study, the thermophoretic velocity of gas mixtures are experimentally measured, and following results are obtained:

1. The thermophoretic velocity depends on the concentration of the gas mixture. For an argon–nitrogen mixture, the velocity increases as the concentration of nitrogen increases; for argon–carbon dioxide and nitrogen–carbon dioxide mixtures, the velocity decreases as the concentration of carbon dioxide increases.
2. The tangential momentum accommodation coefficient of a gas mixture is estimated from experimental results to clarify its dependence on the concentration by assuming the thermal accommodation coefficient at unity. The estimated coefficient is compared with predictions from three methods; it is found that the intermolecular-interaction-based method gives the best prediction among those three methods for all mixture conditions.
3. It is shown that the thermophoretic velocity under a binary gas mixture condition is practically predicted from the intermolecular-interaction-based method.

Acknowledgment

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Table 4
Comparison of coefficients for pure gases.

<table>
<thead>
<tr>
<th>Tested gas</th>
<th>$\alpha_m$</th>
<th>$\alpha_t$</th>
<th>Solid material</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argon</td>
<td>0.931</td>
<td>1.102</td>
<td>Steel spheres (polished surface)</td>
<td>Thomas and Lord (1974)</td>
</tr>
<tr>
<td></td>
<td>1.049</td>
<td>1.161</td>
<td>Steel spheres (rough surface)</td>
<td>Thomas and Lord (1974)</td>
</tr>
<tr>
<td></td>
<td>0.916</td>
<td>1.000</td>
<td>Glass</td>
<td>Sharipov (2004)</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.900</td>
<td>Paraffin</td>
<td>Prodi et al. (2007)</td>
</tr>
<tr>
<td></td>
<td>0.987</td>
<td>1.000</td>
<td>PMMA</td>
<td>This work</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>1.000</td>
<td>0.680</td>
<td>Tungsten</td>
<td>Douglas (1982)</td>
</tr>
<tr>
<td></td>
<td>0.911</td>
<td>1.000</td>
<td>Glass</td>
<td>Sharipov (2004)</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.900</td>
<td>Paraffin</td>
<td>Prodi et al. (2007)</td>
</tr>
<tr>
<td></td>
<td>0.986</td>
<td>1.000</td>
<td>PMMA</td>
<td>This work</td>
</tr>
<tr>
<td>Carbon dioxide</td>
<td>1.000</td>
<td>0.720</td>
<td>Tungsten</td>
<td>Douglas (1982)</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.450</td>
<td>Glass</td>
<td>Li and Davis (1995b)</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.450</td>
<td>Polystyrene latex (PSL) sphere</td>
<td>Li and Davis (1995b)</td>
</tr>
<tr>
<td></td>
<td>0.993</td>
<td>1.000</td>
<td>Glass</td>
<td>Sharipov (2004)</td>
</tr>
<tr>
<td></td>
<td>0.821</td>
<td>1.000</td>
<td>PMMA</td>
<td>This work</td>
</tr>
</tbody>
</table>
References