博士論文内容の要旨／Abstract of Dissertation

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In nonlinear dynamical systems that contain multiple frequencies, quasi-periodic phenomena can emerge. Quasi-periodic phenomena represent non-periodic motions, where the rotation number that corresponds to a ratio of frequencies is an irrational. Quasi-periodic phenomena are commonly observed in various systems, such as biological and engineering systems, and so on. In the field of electric circuits, to derive the generation conditions of the quasi-periodic phenomena is of great importance for applications in designing various practical circuits. It is well known that the synchronization regions that represent periodic motions, where the ratio of frequencies becomes a rational number, exist in the regions-generating quasi-periodic phenomena. The application of synchronization has been widely employed, such as a phase-locked-loops system that generates an output signal whose phase is synchronized to the received signal in electronic communication systems.

Quasi-periodic phenomena have been studied on various types of dynamical systems. At an early stage, nonlinear electric circuits of which governing equations are represented by ordinary differential equations (ODEs) were obtained to clarify the generation mechanism of quasi-periodic phenomena. In general, nonlinear ODEs must be solved by numerical techniques such as averaged method. Since the convergence-time into a steady state of quasi-periodic phenomena is intrinsically long, huge calculation costs are necessary to obtain required accuracy for the analysis. Therefore, most of the detailed studies on quasi-periodic phenomena are targeted to discrete-time dynamical systems. Thus, simpler continuous-time dynamical systems that can obtain the explicit solutions are desired. Many researchers adopted piecewise-linear dynamics to overcome this difficulty. Explicit solutions can be obtained in piecewise-linear branch in these systems. One of the difficulties is that it is necessary to solve the connect condition when the solution strikes the boundary of piecewise-linear branch. However, the connect conditions are expressed by implicit equations. Therefore, many numerical calculations are not only required but also applying the piecewise-linear technique to higher dimensional systems becomes difficult.

In this study, we focus on piecewise-constant oscillators (PWCOs), which are considered as simple continuous-time dynamical systems. The governing equations of PWCOs are represented by piecewise-constant dynamics, and the explicit solutions can be obtained in piecewise-linear regions without solving any implicit equation. Moreover, the solutions on the boundaries of piecewise-constant branches can be explicitly connected. Usually, the analysis of these oscillators arrives at that of the piecewise-linear return map. Therefore,

PWCOs are well suited for theoretical analysis. In addition, PWCOs are quite simple electric circuits. PWCOs consist of linear capacitors, linear resistors, and voltage controlled current sources. The PWCOs can be easily implemented in the laboratory experiment. We propose a calculation algorithm that analyzes the quasi-periodic phenomena with a significantly high resolution similar to that of discrete-time dynamical systems. Based on the algorithm, rigorous bifurcation structures in a two dimensional PWCOs are derived. Such an algorithm can be applicable to a wide range of the family of the PWCOs. Novel bifurcation structures are also observed in a two dimensional PWCOs driven by a periodic external forcing and in a three dimensional PWCOs. Some theoretical results are confirmed by circuit experiments in laboratory.