SIMILARITY ANALYSIS OF AXISYMMETRIC PLUMES AND JETS USING k-ε TURBULENCE MODEL

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The axisymmetric plumes and the axisymmetric jets are investigated analytically. The k-ε turbulence model is adopted in the analysis. Both axisymmetric plumes and jets satisfy the similarity solutions, which are independent on the Reynolds number and the Richardson number. Therefore the similarity Solutions are expected to be applicable to the wide range of hydraulic conditions. The velocity distributions of the plume and the jet obtained herein explain well the experimental results. The distributions of relative density difference predicted for the plume also show a good agreement with the experimental data. The value of the turbulent Schmidt number $\sigma_t = 0.4$ for the axisymmetric plunes is, however, somewhat smaller than the values of $\sigma_t = 0.8\text{-}1.0$ for the inclined wall plumes and for the plane plumes.

Key words: k-ε turbulence model/axisymmetric plumes/axisymmetric jets.

1. INTRODUCTION

Lighter fluid from the point source in the body of the still water flows in the upward direction and forms the axisymmetric plume. Heavier fluid from the point source in the still water forms the downward axisymmetric plume. The axisymmetric plumes are often observed in the natural environment and are important from the engineering aspect. The axisymmetric jets are phenomena similar to the axisymmetric plumes. However, the motive force of the axisymmetric jets is the momentnm flux in the flow direction instead of the buoyancy flux for the axisymmetric plumes.

There are many experimental studies and analytical studies on the axisymmetric plumes and jets. Rouse et al. (1952) have carried out the experiment on the axisymmetric plumes and have obtained data for the velocity and tempartupe distributions. George et al. (1977) have measured the distributions of velocity and temperature of the air plumes by the hot wire anemometer. As the axisymmetric jets, the classical analysis by Tollmien (1926) has been proposed, in which the constant eddy viscosity has been assumed. Reichardt (1944) has carried out the experiment for the axisymmetric jets and compared with the Tollmien's analytical solution.

Recently the flow similar to the axisymmetric plumes and jets are solved by the turbulence model. Sini and Dekeyser (1987) have analyzed the plane plumes and jets to obtain the flow fields by k-ε turbulence model. They solved the partial differential equations numerically. Paullay et al. (1985) have proposed the similarity solutions of velocity distributions of plane and axisymmetric jets by using the k-ε turbulence model. They did not take into account the conservation of momentum flux in the flow direction. Thus their solutions are not complete.

The author (Fukushima, 1988) has shown that the inclined wall plumes have the similarity solutions which satisfy the k-ε turbulence model and the solutions are applicadle to the wide range of flow conditions. The author (Fukushima, 1989) also has obtained the similarity solutions of the plumes and jets and indicated that the solutions explain experimentai data quite well.

The purpose of this study is to obtain the similarity solutions of the axisymmetric plumes and jets using k-ε turbulence model and to discuss the applicability of similarity solutions in comparison with

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the experimental data.

2. BASIC EQUATIONS OF AXISYMMETRIC PLUMES AND JETS

Consider that the axisymmetric plumes whose density is smaller than that of the ambient still fluid flow in the upward vertical direction as shown in figure 1. The x coordinate is taken in the upward direction. The r coordinate is taken in the radial direction. u and v are the velocity components in the x coordinate and the r coordinate, respectively. The radial velocity component v is smaller than the main flow velocity component so that the boundary layer approximation is satisfied. The relative density difference between the plumes and the ambient fluids is small enough to adopt the Boussinesq approximation. This analysis is restricted in the flow establishment region. It is well known that the length of the flow development region is x/d= 4-5 in which d is the diameter of nozzle. Therefore this theory is applicable to x/d>5.

The continuity equation, the momentum equation, the diffusion equation of the scalar quantity which causes the density difference, the equation of the kinetic energy of turbulence and the equation of the viscous dissipation rate of turbulence are written by

$$\frac{\partial u}{\partial x}+\frac{1}{r}\frac{\partial v}{\partial r}=0 \tag{1}$$

$$\frac{\partial u^2}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}(ruv)=Rcg+\frac{1}{r}\frac{\partial}{\partial r}(r\nu_t\frac{\partial u}{\partial r}) \tag{2}$$

$$\frac{\partial}{\partial x}uc+\frac{1}{r}\frac{\partial}{\partial r}(rvc)=\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\nu_t}{\sigma_r}\frac{\partial c}{\partial z}) \tag{3}$$

$$\frac{\partial}{\partial x}uk+\frac{1}{r}\frac{\partial}{\partial z}(rvk)=\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial r})$$
$$+\nu_t(\frac{\partial u}{\partial r})^2-\varepsilon \tag{4}$$

$$\frac{\partial}{\partial x}u\varepsilon+\frac{1}{r}\frac{\partial}{\partial z}(rv\varepsilon)=\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\nu_t}{\sigma\varepsilon}\frac{\partial \varepsilon}{\partial z})$$
$$+c_{1\varepsilon}\frac{\varepsilon}{k}\nu_t(\frac{\partial u}{\partial r})^2-c_{2\varepsilon}\frac{\varepsilon^2}{k} \tag{5}$$

where g is the gravity acceleration, c is the concentration of scalar quantity, and R is the ratio of the relative density difference to the concentration or temperature. The eddy viscosity of the turbulence ν_t is defined by

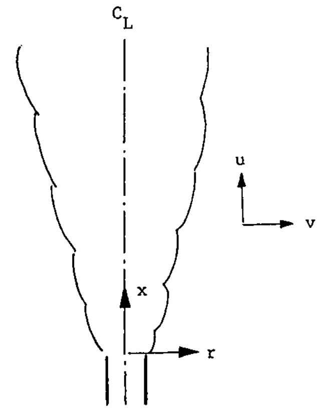

Figure 1 Definition sketch of axisymmetric plumes

$$\nu_t=c_\mu k^2/\varepsilon \tag{6}$$

The values of numericai constant appearing in the model are given by

$$c_\mu=0.09 \quad \sigma_k=1.0 \quad \sigma_\varepsilon=1.3 \quad c_{1\varepsilon}=1.44$$
$$c_{2\varepsilon}=1.92 \tag{7}$$

It is noted that in case of the axisymmetric jets, the gravity term in equation (2) is zero.

Introducing the stream function ψ, velocity components satisfy the following relations.

$$u=\frac{1}{r}\frac{\partial\psi}{\partial r} \tag{8 a}$$

$$v=-\frac{1}{r}\frac{\partial\psi}{\partial x} \tag{8 b}$$

3. SIMILARITY SOLUTIONS OF AXISYMMETRIC PLUMES

In order to obtain the similarity solutions of the axisymmetric plumes, the similarity variable η is introduced. The stream function ψ, the concentration c, the kinetic energy of turbulence k, and dissipation rate of turbulence ε are expressed by the following equations.

$$\eta=arx^{-l} \tag{9 a}$$

$$\psi=bx^mF(\eta) \tag{9 b}$$

$$c=c_* x^nG(\eta) \tag{9 c}$$

$$k=dx^pK(\eta) \tag{9 d}$$

$$\varepsilon=ex^qE(\eta) \tag{9 e}$$

where a, b, c_*, d and e are the coefficients which should be determined below, and the powers of x, l, m, n, p and q also should be determined below. F, G, K and E are the similarity functions for ψ, c, k and ε, respectively. The velocity components u and v are obtained from equations (8) and (9 b) as follows:

$$u=\frac{1}{r}\frac{\partial\psi}{\partial r}=\frac{1}{r}\frac{\partial}{\partial r}\{bx^m F(\eta)\}=\frac{a}{x^l\eta}bx^m\frac{\partial\eta}{\partial r}F'$$

$$=a^2bx^{m-2l}\frac{F'}{\eta} \qquad (10\text{ a})$$

$$v=-\frac{1}{r}\frac{\partial\psi}{\partial x}=-r\frac{\partial}{\partial x}\{bx^m F(\eta)\}$$

$$=-abx^{m-l-1}(m\frac{1}{\eta}F-alF') \qquad (10\text{ b})$$

where the prime ($'$) means the ordinary derivative with respect to η. The non-dimensional eddy viscosity and its derivative are defined by

$$\nu_{t*}=c_\mu\frac{K^2}{E} \qquad (11\text{ a})$$

$$\nu'_{t*}=c_\mu\left\{\frac{2KK'}{E}-\frac{K^2E'}{E^2}\right\} \qquad (11\text{ b})$$

Substituting equations (8) - (11) into equations (2) - (5), the following set of equations is derived after some manipulations.

$$x^{2m-4l-1}\{(m-2l)\frac{1}{\eta^2}F'^2+m\frac{1}{\eta^3}FF'-m\frac{1}{\eta^2}FF''\}$$

$$=\frac{Rgc_*}{a^4b^2}x^nG+\frac{d^2}{be}x^{2p-q+m-4l}\{\nu'_{t*}(\frac{1}{\eta}F')'$$

$$-\eta\,\nu_{t*}(\frac{1}{\eta}F')'+\frac{1}{\eta}\nu_{t*}F'''\} \qquad (12)$$

$$x^{m+n-2l-1}\frac{1}{\eta}(nF'G-mFG')$$

$$=\frac{d^2}{be}x^{2p-q+n-2l}\frac{1}{\sigma_t}(\nu_{t*}\frac{G'}{\eta}+\nu'_{t*}G'+\nu_{t*}G'') \qquad (13)$$

$$x^{m+p-2l-1}\frac{1}{\eta}(pF'K-mFK')$$

$$=\frac{a^4bd}{e}{}^{2m-6l+2p-q}\nu_{t*}(\frac{1}{\eta}F')'^2+\frac{d^2}{be}x^{3p-q-2l}$$

$$\frac{1}{\sigma_k}(\frac{1}{\eta}\nu_{t*}K'+\nu_{t*}K'')-\frac{e}{a^2bd}x^qE \qquad (14)$$

$$x^{m+q-2l-1}\frac{1}{\eta}(qF'E-mFE')$$

$$=\frac{a^4bd}{e}x^{2m-6l+p}c_{1\varepsilon}\frac{E}{K}\nu_{t*}(\frac{1}{\eta}F')'^2$$

$$-\frac{e}{a^2bd}x^{2q-p}c_{2\varepsilon}\frac{E^2}{K}$$

$$+\frac{d^2}{be}x^{2p-2l}\frac{1}{\sigma_\varepsilon}\{\frac{1}{\eta}\nu_{t*}E'+\nu'_{t*}E'+\nu_{t*}E''\} \qquad (15)$$

In the case of the axisymmetric plumes, the buoyancy flux in the vertical direction, Q_c, is conserved. Integration of equation (3) leads to

$$Q_c=\int_0^\infty 2\pi ruc\,dr=2\pi bc_*x^{m+n}\int_0^\infty F'G\,d\eta \qquad (16)$$

Q_c is a constant in the flow direction.

Under the conditions that equations (12) to (16) have the similarity solutions, the powers of x in each equation are equal. These conditions are

$$2m-4l-1=n=2p-q+m-4l \qquad (17\text{ a})$$

$$m+n-2l-1=2p-q+n-2l \qquad (17\text{ b})$$

$$m-2l+p-1=2m-6l+2p-q=3p-p-2l=q \qquad (17\text{ c})$$

$$m+q-2l-1=2m-6l+p=2q-p=2p-2l \qquad (17\text{ d})$$

$$m+n=0 \qquad (17\text{ e})$$

The powers of x are determined from the above equations as follows:

$$l=1 \quad m=5/3 \quad n=-5/3$$

$$p=-2/3 \quad q=-2 \qquad (18)$$

In order to determine coeficients a, b, c_*, d and e in equations (9), coefficients in equations (12) to (16) should be set to be unity as follows:

$$\frac{Rgc_*}{a^4b^2}=\frac{d^2}{be}=\frac{a^4bd}{e}=\frac{e}{a^2bd}=\frac{Q_c}{bc_*}=1 \qquad (19)$$

Coefficients a, b, c_*, d and e are determined by equations (19), as follows:

$$a=1 \qquad (20\text{ a})$$

$$b=(RgQ_c)^{1/3} \qquad (20\text{ b})$$

$$c_*=Q_c^{2/3}/(Rg)^{1/3} \qquad (20\text{ c})$$

$$d=(RgQ_c)^{2/3} \qquad (20\text{ d})$$

$$e=RgQ_c \qquad (20\text{ e})$$

Substituting equation (19) into equations (12)-(15), the differential equations for F, G, K and E become

$$\frac{1}{3}\frac{1}{\eta^2}F'^2-\frac{5}{3}\frac{1}{\eta^3}FF'+\frac{5}{3}\frac{1}{\eta^2}FF''+G$$

$$+\nu'_{t*}(\frac{1}{\eta}F')'-\frac{1}{\eta}\nu_{t*}(\frac{1}{\eta}F')'+\frac{1}{\eta}\nu_{t*}F'''=0 \qquad (21)$$

$$\frac{5}{3}\frac{1}{\eta}(FG)'+\frac{1}{\sigma_t}\frac{1}{\eta}(\eta\nu_{t*}G')'=0 \qquad (22)$$

$$\frac{1}{\eta}\{\frac{2}{3}F'K+\frac{5}{3}FK'\}+\nu_{t*}(\frac{1}{\eta}F')'^2$$

$$+\frac{1}{\sigma_k}\frac{1}{\eta}(\eta\nu_{t*}K')'-E=0 \tag{23}$$

$$\frac{1}{\eta}\{2F'E+\frac{5}{3}FE'\}+c_{1\varepsilon}\frac{E}{K}\nu_{t*}(\frac{1}{\eta}F')'^2$$

$$\frac{1}{\sigma_\varepsilon}\frac{1}{\eta}(\eta\nu_{t*}K')'-c_{2\varepsilon}\frac{E^2}{K}=0 \tag{24}$$

Substituting equations (18) and (20) into (9), the similarity variable η, the stream function ψ, the kinetic energy of turbulence k, the viscous dissipation rate ε, the velocity in the flow direction u and the eddy viscosity ν_t are expressed by

$$\eta=r/x \tag{25 a}$$

$$\psi=(RgQ_c)^{1/3}x^{5/3}F \tag{25 b}$$

$$c=\frac{Q_c^{2/3}}{(Rg)^{1/3}}x^{-5/3}G \tag{25 c}$$

$$k=(RgQ_c)^{2/3}x^{-2/3}K \tag{25 d}$$

$$\varepsilon=(RgQ_c)x^{-2}E \tag{25 e}$$

$$u=(RgQ_c)^{1/3}x^{-1/3}\frac{1}{\eta}F' \tag{25 f}$$

$$\nu_t=(RgQ_c)^{1/3}x^{2/3}\nu_{t*} \tag{25 g}$$

Next we consider the boundary conditions. All variables are symmetrical at the axis (r=0), and the radial velocity component becomes zero at the axis. Thus,

$$\frac{\partial u}{\partial r}=\frac{\partial c}{\partial r}=\frac{\partial k}{\partial r}=\frac{\partial \varepsilon}{\partial r}=v=0 \tag{26}$$

Non-dimensional forms of these conditions are

$$F=(F'/\eta)'=G'=K'=E'=0 \tag{27}$$

All variables becomes zero at the outer edge of the plumes, r/x = 0.18, which are

$$u=c=k=\varepsilon=0 \tag{28}$$

Non-dimensional expressions of these conditions becomes

$$F'=G=K=E=0 \tag{29}$$

As shown above, the characteristics of the axisymmetric plumes are described by the set of the differential equations (21) to (24) with the boundary conditions (27) and (29). It is seen from these equations that the similarity solutions are independent on the Reynolds number and the Richardson number. All dimensional variables are determined by the bouyancy flux as shown in equation (25).

4. SIMILARITY SOLUTIONS OF AXISYMMETRIC JETS

The same procedure in the previous section is used to obtain the similarity solutions of the axisymmetric jets. The similarity variable and similarity functions are expressed by equation (9). In the case of the jet, the gravity term of equation (12) is omitted as follows:

$$x^{2m-4l-1}\{(m-2l)\frac{1}{\eta^2}F'^2+m\frac{1}{\eta^3}FF'-m\frac{1}{\eta^2}FF''\}$$

$$=x^{2p-q+m-4l}\frac{d^2}{be}\{\nu'_{t*}(\frac{1}{\eta}F')'$$

$$-\frac{1}{\eta}\nu_{t*}(\frac{1}{\eta}F')'+\frac{1}{\eta}\nu_{t*}F''') \tag{30}$$

The momentum flux is defined as

$$M=\int_0^\infty 2\pi ru^2dr=2\pi a^2b^2x^{2m-2l}\int_0^\infty\frac{1}{\eta}F'^2d\eta \tag{31}$$

In order to find the similarity solutions for the jet, the powers of x in equation (30) and (31) are equal in each equation.

$$2m-4l-1=2p-q+m-4l \tag{32 a}$$

$$2m-2l=0 \tag{32 b}$$

The powers of x are determined from the above equations and equations (17 b) to (17 d) as follows.

$$l=1\ m=1\ n=-1\ p=-2\ q=-4 \tag{33}$$

The coefficient a, b, c_*, d and e are obtained from the condition that the coefficients in equations (30) and (31) in addition to equations (13) to (16) are set to be unity. Thus,

$$\frac{d^2}{be}=\frac{a^4bd}{e}=\frac{e}{a^2bd}=\frac{M}{a^2b^2}=\frac{Q_c}{bc_*}=1 \tag{34}$$

The coefficients a, b, c_*, d and e are determined as follows:

$$a=1 \tag{35 a}$$

$$b=M^{1/2} \tag{35 b}$$

$$c_*=Q_c/M^{1/2} \tag{35 c}$$

$$d=M \tag{35 d}$$

$$e=M^{3/2} \tag{35 e}$$

Substituting equation (34) into equation (30) and equations (13) to (18), the following ordinary differential equations for the axisymmetric jet are obtained.

$$\frac{1}{\eta^2}F'^2-\frac{1}{\eta^3}FF'+\frac{1}{\eta^2}FF''$$

$$+\nu'_{t*}(\frac{1}{\eta}F')'-\frac{1}{\eta}\nu_{t*}(\frac{1}{\eta}F')'+\frac{1}{\eta}\nu_{t*}F'''=0 \tag{36}$$

$$\frac{1}{\eta}(FG)'+\frac{1}{\sigma_t}\frac{1}{\eta}(\eta\nu_{t*}G')'=0 \tag{37}$$

$$\frac{1}{\eta}(2F'K+FK')+\nu_{t*}(\frac{1}{\eta}F')'^2$$
$$+\frac{1}{\sigma_k}\frac{1}{\eta}(\eta\nu_{t*}K')'-E=0 \tag{38}$$

$$\frac{1}{\eta}(4F'E+FE')+c_{1\varepsilon}\frac{E}{K}\nu_{t*}(\frac{1}{\eta}F')'^2$$
$$-c_{2\varepsilon}\frac{E^2}{K}+\frac{1}{\sigma_\varepsilon}\frac{1}{\eta}(\eta\nu_{t*}E')'=0 \tag{39}$$

The similarity variable and similarity functions are obtained from substitution of equation (33) and (35) into equation (9) as follows:

$$\eta=r/x \tag{40 a}$$
$$\psi=M^{1/2}xF \tag{40 b}$$
$$c_*=\frac{Q_c}{M^{1/2}}x^{-1}G \tag{40 c}$$
$$k=Mx^{-2}K \tag{40 d}$$
$$\varepsilon=M^{3/2}x^{-4}E \tag{40 e}$$
$$u=M^{1/2}x^{-1}\frac{1}{\eta}F' \tag{40 f}$$
$$\nu_t=M^{1/2}\nu_{t*} \tag{40 g}$$

The same boundary conditions as the plumes are used for the axisymmetric jets as equations (27) and (29).

The similarity solutions for the axisymmetric jets are obtained from the solutions of set of the differential equations (36) to (39) with the boundary conditions (27) and (29). The same equations as equations (36). (38) and (39) are derived by Paullay et al.(1985). They, however, could not determine coefficients described by equation (40), because they did not use equation (31).

5. RESULTS OF NUMERICAL SOLUTIONS AND DISCUSSIONS

The similarity solutions obtained by the numerical calculation for the axisymmetric plumes are shown in figures 2 a-2 e. The turbulent Schmidt number σ_t is chosen as the parameter in these figures. The similarity solution of the velocity distributions changes weakly by the value of σ_t. On the contrary, the distribution of the relative density difference changes remarkably by the value of σ_t. The nondimensional kinetic energy of turbulence and the viscous dissipation rate has a peak around $\eta=0.06$. The same results have been observed in the case of the plane plumes (Fukushima,1989). The eddy viscosity (figure 2e) does not have a clear peak and is almost a constant near the central region. The eddy viscosity of the plane plumes has a clear peak and depress near the central region. The difference between the axisymmetric plumes and the plane plumes is considered to be caused by the different turbulent structure between two types of flows.

Comparisons between the numerical solutions and the experimental data obtained by George et al. (1977) and Rouse et al. (1952) are made in figure 3 a and 3b. The value of the parameter σ_t in the numerical solution is chosen to be 0.4. George et al.'s the experimetal data of the nondimensional velocity are larger in the outer region of the plume compared with the numerical results. Rouse et al.'s experimental data are larger in the central region. The similarity solution of distributions of the relative density difference (figure 3 b) agrees well with both George et al.'s and Rouse et al.'s experimental data. However, The value of $\sigma_t=0.4$ is somewhat small compared with $\sigma_t=0.8$ to 1.0 for the inclined wall plume (Fukushima, 1988) and for the plane plume (Fukushima, 1989). The more precise experiment and analysis are needed to clarify the effect of σ_t. As shown in figure 3 a, the theoretical velocity profile is large in the central region and small in the outer region compared with the Gearge et al.'s experimental data. Using the value of $\sigma_t=0.8$-1.0, the theoretical velocity profile explain well the experimental results.

The numerical results of the similarity solutions of the axisymmetric jets are illustrated in figures 4 a-4 e. In this case, only the concentration is dependent on σ_t, the others are independent on σ_t. It is seen from these figures that the velocity and concentration are approximately described by the Gaussian distribution. The distributions of the kinetic energy of turbulence, the viscous dissipation rate and the eddy viscosity have no peak as shown in

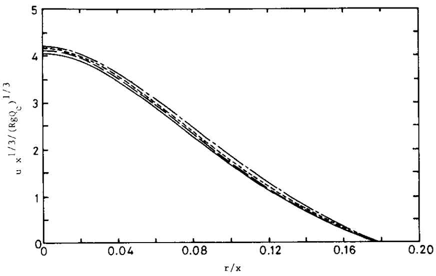

Fig. 2 a velocity

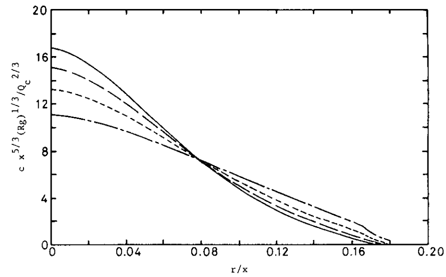

Fig. 2 b relative density difference

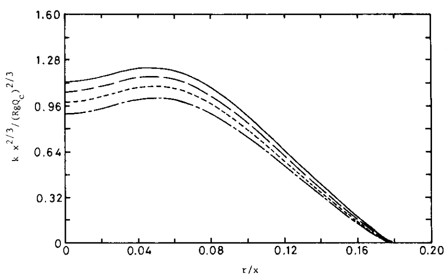

Fig. 2 c kinetic energy of turbulence

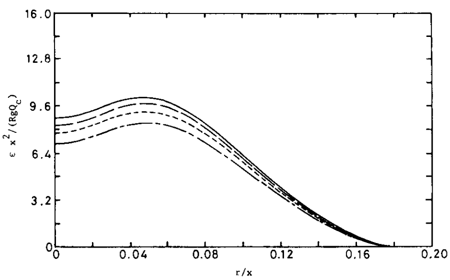

Fig. 2 d viscous dissipation rate of turbulence

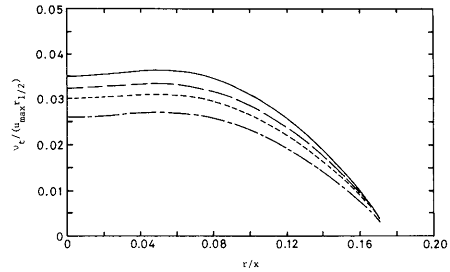

Fig. 2 e eddy viscsity

Figure 2 Similarity solutions of the axisymmetric plumes. Solid line indicates $\sigma_t=1.0$, long dash line indicates $\sigma_t=0.8$, dot line indicates $\sigma_t=0.6$ and dash and dot line indicates $\sigma_t=0.4$.

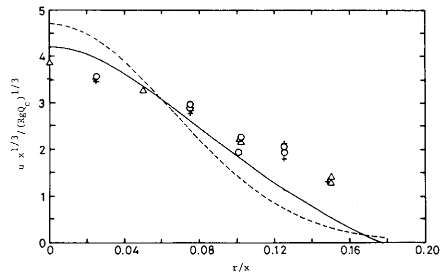

Figure 3 a Comparison of similarity solutions of velocity distribution of the axisymmetric plumes with experimental data. Solid line indicates the numerical solution of $\sigma_t=0.4$ and dash line indicates Rouse et al. (1952)'s experimental results. Symbols are George et al. (1977)'s experimental data, where open circles indicate x/D = 16, triangles indicate x/D=12 and crosses indicate x/D = 8, and D is the diameter of the inlet.

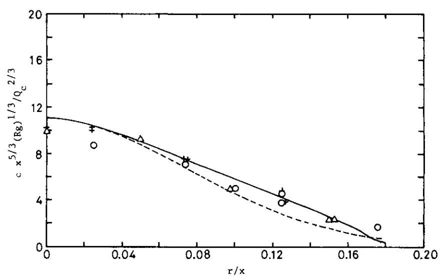

Fig. 3 b relative density difference

Figure 3 b Comparison of similarity solutions of distribution of relative density difference of the axisymmetric plumes with experimental data. Symbols are the same as figure 3 a.

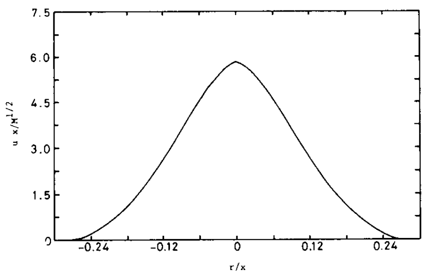

Fig. 4 a velocity

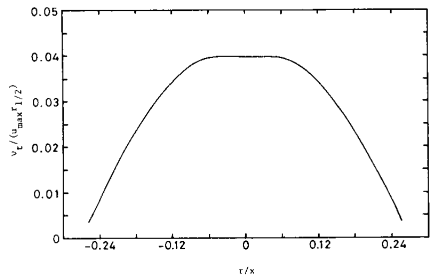

Fig. 4 e eddy viscosity

Figure 4 Similarity solutions of the axisymmetric jets. Solid line indicates $\sigma_t = 1.0$, long dash line indicates $\sigma_t = 0.8$, dot line indicates $\sigma_t = 0.6$.

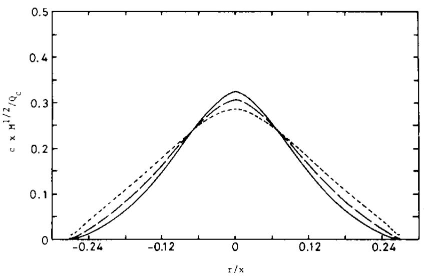

Fig. 4 b concentration

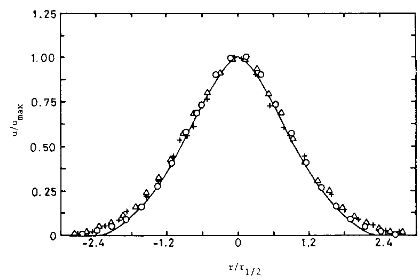

Figure 5 Comparison of similarity solutions of velocity distribution of the axisymmetric jets with experimental data. Solid line indicates the numerical solution. Symbols are Reichardt (1944)'s experimental data, where open circles indicate x = 20 cm, triangles indicate x = 26 cm and crosses indicate x = 45 cm.

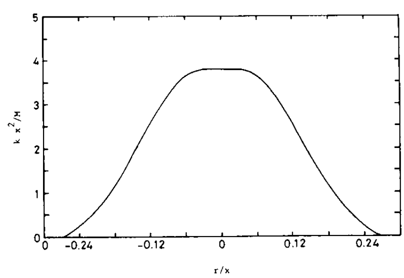

Fig. 4 c kinetic energy of turbulence

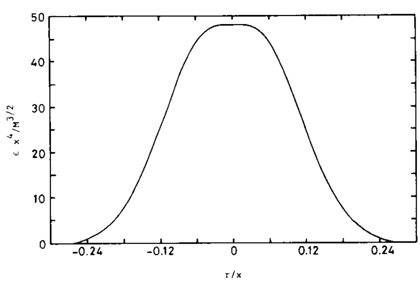

Fig. 4 d viscous dissipation rate of turbulence

figures 4 c, 4 d and 4 e are very flat near the central region. Thus the structure of turbulence of the axisymmetric jets is different from those of the plane jets and the axisymmetric plumes.

Comparison between the numerical solutions and Reichardt (1944)'s experimental data is made in figure 5. The velocity is normalized by the maximum velocity and r coordinate is normalized by the half width of the maximum velocity. The numerical solution obtained herein explains well the experimental results in the whole region.

6. CONCLUSIONS

The axisymmetric plumes and the axisymmetric jets are investigated analytically. The similarity solutions of both flows are obtained using the k-ε turbulence model. The numerical solutions are obtained quite easily to solve the set of ordinary differential equations. The similarity solution of the axisymmetric plumes with $\sigma_t = 0.4$ explains well the experimental results of the velocity and temperature distribution. This value of σ_t seems to be rather small compared with those obtained from the other flow. The velocity distribution of the axisymmetric jets obtained by the similarity analysis agrees quite well with the experimental data.

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